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APPLIED MECHANICS REVIEWS

VOL. 4, NO. 3 MARTIN GOLAND *Editor* MARCH 1951

A REVIEW OF AEROELASTICITY

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ALL aeroelastic problems involve the feed-back circuit:

elastic deformations $\rightleftarrows$ aerodynamic loads

where the arrows signify *affect*. In the general dynamic case, inertia and, perhaps, frictional forces contribute to the loads. The underlying mathematical equations are always linearized; they are homogeneous in stability problems *S*, and non-homogeneous in response problems *R*. The aerodynamic forces do not respond instantaneously to the motion of the system; the circulatory parts are subject to "aerodynamic lag," as indicated by the Wagner-Küssner-Theodorsen theories.

Among the important aeroelastic problems are:

S1 Flutter, the "classical" aeroelastic stability problem controlled by equations including inertia terms and taking aerodynamic lag into account.

S2 Wing divergence, that is, static aeroelastic stability. Inertia loads and aerodynamic lag vanish.

S3 Aeroelastic effects on flight stability, which can be expressed by "slowly varying" solutions which allow for neglect of inertia forces due to variation of elastic deformations and, generally, of aerodynamic lag.

R1 Buffeting, that is, the tailplane response to eddying flow in the wake of the wing close to stalling.

R2 Control reversal and redistribution of aerodynamic load due to structural distortion (response to controls and to angle of attack), represented by stationary solutions of the mathematical equations, inertia loads, and aerodynamic lag vanishing.

R3 Aeroelastic effects on maneuverability, depending on slowly varying solutions, again allowing the simplifications specified under *S3*.

R4 Transient oscillations generated by gusts, a general dynamical problem.

In all aeroelastic considerations the elasticity of the system is simplified, generally to a quite elementary model. Chordwise sections of wings and control surfaces are taken to be perfectly rigid, usually a justifiable assumption. The deformations are then broken down into bending and torsion in accordance with elementary, straight beam deformation theory. A straight elastic axis is presumed, the bending moment is computed as EIz'', the torsional moment as $GJ\varphi'$. This procedure is not, however, unconditionally adequate. For large airplanes, shear deflection and shear lag appear to contribute to the elastic properties (1).* For certain wing designs, differential bending affects the behavior in torsion (2). The most common and most important complication, however, is the occurrence of appreciable cross-stiffnesses, i.e., elastic couplings between bending and torsion, which occur even in unswept wings in the absence of a straight elastic axis (due, for example, to cutouts or other design discontinuities). Except for swept wings (3, 4), where the problem is acute and still not definitely solved, this complication has received little attention.

In usual practice, the computed elastic properties and inertia distributions of an airplane are checked experimentally by means of a resonance test of the complete airplane. It is generally suggested that agreement is easily obtained by applying slight corrections to computed stiffnesses. Frequently, this is far from true. Often, agreement not only cannot be attained without introducing cross-stiffnesses, but, in complicated cases (such as when there are many natural frequencies, partly close together) it may even be difficult to deduce appropriate diagonal stiffnesses from the test results. A good paper on this subject would certainly be useful. The results of resonance tests may, themselves, also be far from perfect. In this respect, more elaborate methods have been proposed recently (5) which should result in important improvements.

For straight wings, the distributed aerodynamic forces required for the solution of the stationary problems of aeroelasticity *S2*, *R2*, are conventionally based either on the quite elementary "strip theory" or on Prandtl's integral equation. A remarkable new method for solving this latter equation has recently been developed by Sears (6). It offers particular advantages if the equation is connected with aeroelastic *R2* problems. Other progress (7) concerns mainly the consistent reduction of the mathematics to easily mechanizable matrix schemes.

In the case of swept wings, Weissinger's method is generally used to obtain the distribution of aerodynamic forces. This method reproduces the spanwise lift distribution accurately; however, it should not be considered as entirely adequate because it gives no information regarding the chordwise shifts in local center of pressure occurring under the influence of sweepback, in particular in the center and tip regions. Reissner (8) has proposed methods which take this effect into consideration. A final solution of the incompressible, aerodynamic problem of the thin swept wing has certainly not yet been reached.

In the subsonic region, the effects of compressibility can be traced by the simple Prandtl-Glauert rule. Transonic and super-

EDITOR'S NOTE: The third in a series of articles, contributed to APPLIED MECHANICS REVIEWS by international authorities, surveying important topics in applied mechanics.

* Numbers in parentheses refer to Bibliography at end of the paper.

sonic cases need not be considered here because, as yet, too little has been published on *R2* problems.

As for the quasisteady *S3* and *R3* problems, the usual procedures for obtaining stability derivatives, etc., are added to the methods already noted in the earlier discussion.

In flutter calculations not involving compressible flow, the elementary strip-theory adaptation of the Küssner-Theodorsen aerodynamic coefficients to finite span can, if necessary, be now replaced for straight wings by acceptable correction methods (9, 10). For swept wings, where the corrections may be expected to be still more important, no suitable theory is available. The opportunities for accounting for compressibility in the subsonic régime are still inadequate, existing tables of coefficients being tentative and incomplete. Provisional correction procedures of a Prandtl-Glauert type can, at most, have some empirical usefulness. New fairly extensive tables based on a theory of Timman (11) are being prepared in Holland. Very extensive tables of aerodynamic coefficients of oscillating wings in two-dimensional supersonic flow have been published recently in the United States (12). Similar tabulations are being prepared in France (13).

Available experimental checks of the Küssner-Theodorsen theory, used so extensively all over the world, are still fragmentary [e.g., Cicala (14)]. A recent paper by Drescher (15) reporting on measurements of aerodynamic coefficients of a wing with an oscillating control surface in two-dimensional incompressible flow is, therefore, of considerable interest.

The aerodynamics underlying the buffeting phenomena in problem *R1* is still fairly obscure.

Progress in flutter analysis itself largely concerns the development of methods allowing for the treatment of complicated systems with many degrees of freedom, e.g. (16). More and more, calculation procedures are being converted into readily surveyable and mechanizable pure matrix schemes, and direct iteration methods for flutter modes and frequencies are becoming more important. An attempt has been made to improve the mathematical background of flutter analysis [complicated by the non-self-adjointness of the basic equations, as indicated by Wielandt (17)] and to give a critical account of the numerous Ritz-Galerkin type and iteration methods applicable to flutter problems (18). Analog machines and commercial punched-card equipment have been employed in flutter calculations. Both are useful, but not particularly convenient. Electronic and relay machines, however, should allow considerable expansion in the scope of flutter calculations.

Extensive calculations have been made for highly flutter-sensitive wing-control-tab systems (19, 20). Flutter of swept wings constitutes a difficult problem, at least in principle, both the elastic and the aerodynamic properties being complicated and not covered by entirely satisfactory theories, as stated previously. A combined theoretical and experimental approach has just been published by the National Advisory Committee for Aeronautics (21).

A report by Smilg (22) deals with the question of aileron oscillations at transonic speeds. On the whole, information applying to this singular speed range is regrettably small.

The application of the experimental techniques of flutter analysis has been facilitated by a valuable report (23) from a group experienced in this type of work, on the design of flutter models suitable for wind-tunnel analysis of the flutter characteristics of an airplane. Experimental work providing important information on the various flutter modes of tail systems has been carried out in England (24).

The aeroelastic *S3*, *R2*, and *R3* problems have attained considerable importance for high-speed aircraft (25). For swept wings, the *R2* problem involves major design criteria, and a good deal of work has been directed to this type of wing (26, 27). Since flutter is usually suppressible by suitable mass decoupling (mass balance), the aeroelastic *S2*, *S3*, *R2*, and *R3* phenomena tend to become determinative for the stiffnesses. They may be of still greater importance in the supersonic range. An exhaustive analysis of these problems is hampered by the complexity of the required calculations.

In conclusion, attention is called to the fact that many of the references mentioned in this review are used mainly as illustrative material, a complete bibliography not being intended.

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3. Levy, Samuel, Computation of influence coefficients for aircraft structures with discontinuities and sweepback, AMR 1, Rev. 79.
4. Hemp, W. S., On the application of oblique coordinates to problems of plane elasticity and sweptback wing structures, AMR **3,** Rev. 2207.
5. Kennedy, C. C., and Pancu, C. D. P., Use of vectors in vibration measurement and analysis, AMR **1,** Rev. 25.
6. Sears, W. R., A new treatment of the lifting-line wing theory with applications to rigid and elastic wings, AMR 1, Rev. 1658.
7. Pines, S., A unit solution for the load distribution of a nonrigid wing by matrix methods, AMR **3,** Rev. 2465.
8. Reissner, E., Note on the theory of lifting surfaces, AMR **4,** Rev. 760.
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10. Reissner, E., and Stevens, J. E., Effect of finite span on the air load distributions for oscillating wings—II, AMR **1,** Rev. 887.
11. Timman, R., and van de Vooren, A. I., Theory of the oscillating wing with aerodynamically balanced control surface in a two-dimensional subsonic compressible flow, Rep. F 54, Nat. Aero. Res. Inst., Amsterdam, 1949.
12. Huckel, V., and Darling, B. J., Tables of wing aileron coefficients of oscillating air forces for two-dimensional supersonic flow, Rev. 1310 in this issue.
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14. Cicala, P., Ricerche sperimentale sulle azione aerodinamiche sopra l'ala oscillante, l'Aerotecnica **XXI,** 1, 1941.
15. Drescher, H., Eine experimentelle Bestimmung der aerodynamischen Reaktionen auf einem Flügel mit schwingendem Ruder, Öst. Ingen.-Arch. **IV,** 3–4, p. 270, 1950.
16. Duncan, W. J., Flutter of systems with many degrees of freedom, AMR **2,** Rev. 114.
17. Wielandt, H., Das Iterationsverfahren bei nichtselbstadjungierten linearen Eigenwertaufgaben, Math. Z. **50,** p. 93, 1944.
18. Greidanus, J. H., and van de Vooren, A. I., Mathematical principles of flutter analysis, AMR **3,** Rev. 1355.
19. Frazer, R. A., and Jones, W. P., Wing-aileron-tab flutter, I–V. ARC Reps. 5668 and 6290, 1942.
20. van de Vooren, A. I., and Hofsommer, D. J., Binary aileron-tab flutter, AMR **3,** Rev. 2756.
21. Barmby, J. G., et al., Study of effects of sweep on the flutter of cantilever wings, Rev. 1309 in this issue.
22. Smilg, B., The prevention of aileron oscillations at transonic speeds, Sixth int. Congr. appl. Mech., Paris, 1946.
23. Beckley, L. B., Report on the design of sectional-type

flutter and dynamic models of aircraft structures, Rep. Mass. Inst. Tech. 7493, 1948.

24. Scruton, C., Experiments on tail flutter, Aero. Res. Coun. Rep. Mem., R. & M. 2323, 1948.

25. Collar, A. R., Aeroelastic problems at high speed, AMR **1**, Rev. 161.

26. Templeton, Haydn, Control reversal effects on swept-back wings, AMR **3**, Rev. 1358.

27. Pai, S. I., and Sears, W. R., Some aeroelastic properties of swept wings, AMR **2**, Rev. 676.

Communications

Concerning AMR 3, Rev. 1668 of W. S. Hemp, The theory of flat panels buckled in compression.

It is stated that the results of R. & M. 2128 are smaller than the "exact" results of S. Levy [*Nat. adv. Comm. Aero. Rep.* **737**]. This does not surprise me, since in Levy's solution the edges of the plate are constrained to remain straight, whereas in my solution, given in the R. & M., the edges are "free"—for boundary conditions "parallel" to the plane of the plate. The results of R. & M. 2128 are "exact" for the problem considered.

W. S. Hemp, England

Correction to AMR 4, Rev. 584

At beginning of review, please read "Picone method" for "Ticone method."

Theoretical and Experimental Methods

(See also Revs. 1004, 1027, 1058, 1089, 1235, 1283, 1351)

973. Milne, W. E., Note on the Runge-Kutta method, *J. Res. nat. Bur. Stands.* **44,** 5, 549–550, May 1950.

The Runge-Kutta method for the numerical solution of differential equations has well-known advantages: (1) No special devices are required for starting the computation; (2) length of the step can be modified at any time in the course of the computation without additional labor. Its serious disadvantages often drive one to seek other schemes: (a) The process does not contain in itself any simple method either for estimating the error or for detecting mistakes in computation; (b) each step requires four substitutions into the differential equation (an amount of labor often excessive). The examples usually cited show the method in an unreasonably favorable light. Author proposes a different procedure also of the fourth degree in the increment, h, and one with a simple accompanying control. Instances are given where the new procedure is obviously preferable. The value of y_{n+1} is computed from those of y_{n-1}, y_n', and $\delta^2 y_n'$ (the second central difference of y_n'). Other methods, not mentioned by author, are in use. Albert A. Bennett, USA

974. Vand, V., A mechanical x-ray structure-factor calculating machine, *J. sci. Instrum.* **27,** 10, 257–261, Oct. 1950.

Paper describes a mechanical analog computer for x-ray structure-factor calculations. Simultaneous summation of the Fourier terms is achieved by a pulley system first devised by Kelvin. Of particular interest is the toothless ratchet mechanism which is intrinsically more accurate than the equivalent geared system used in a previous machine. Friction build-up in the pulley adding mechanism limits maximum capacity of machine, which in its present state can deal with structures containing up to 48 atoms per unit cell having a center of symmetry.

Machine has been devised to handle the method of steepest descents but has not proved altogether satisfactory on account of slow convergence of the method. Author indicates that it is not yet clear whether this is due to construction of the machine or to method of stating the problem. A. Porter, Canada

975. Willers, Fr. A., Automatic computers (in German), *Arch. tech. Messen*, no. 169, T16–18, Feb. 1950.

A survey including modern, American, English, French, and German developments, and some basic concepts of the logical design of automatic computers. Ed.

976. Middleton, W. E. Knowles, The curve computer, *Canad. J. Res.* **28,** sec. F, 8, 324–332, Aug. 1950.

An electromechanical instrument has been developed which adds, subtracts, multiplies, or divides the ordinates of two curves in Cartesian coordinates, plotting the resulting locus. This uses the Wheatstone bridge principle, and is semiautomatic; the curves are followed manually, but a phase-sensitive servo system keeps the bridge in balance. Attachments are provided for determining the area under the resulting curve, and also for performance of certain logarithmic operations of interest in photometry. In the ordinary use of the computer, the accuracy is about 0.3% of full scale or better. From author's summary

977. Bouthillon, M. L., Big calculating machines, I, II (in French), *Mémor. Artill. fr.* **24,** nos. 1, 2, 221–247, 251–324, 1950.

Author surveys the development of calculating machines including the most recent types and explains in detail physical principle on which each machine is based and the mode of operation.

First part describes calculating machines and differential analyzers which are operated by purely mechanic, electromagnetic, or electric-control devices (integrator of V. Bush, automatic sequence controlled calculator of Harvard University, and machine of the Bell Laboratories). Second part deals with use of thermionic valves for calculating machines: the explanation of the basic electronic control device (in USA called flip-flop) and its manifold applications. Third part describes the mode of operation and construction of the big electronic calculating machines (ENIAC, EDVAC, UNIVAC, machines of the Raytheon Company, of Massachusetts Institute of Technology, of International Business Machines Corporation, and of Institute for Advanced Study in USA; the EDSAC, the machine in the National Physical Laboratory, and other projects in England).

Manfred Shafer, Germany

978. Korn, Granino A., Stabilization of simultaneous equation solvers, *Proc. Inst. Radio Engrs.* **37,** 1000–1002, 1949.

Paper is concerned primarily with simultaneous linear equation solving machines in which the error in ith equation is used to change the ith unknown directly through a feed-back amplifier. It is assumed that the system of equations is positive definite. Problem of designing amplifiers so that a system of n equations will be stable is reduced to that of designing an amplifier for a single equation $ax + b = 0$, where a may be a complex number with a positive real part. "The fundamental importance of the last-mentioned theorem lies in the fact that it reduces the design of amplifiers for the complicated multiple-loop feed-back system to design of one simple feed-back amplifier, so that stability criteria known from experience or a Nyquist analysis may be applied. Described analysis can be extended to cases in which various

amplifiers are not exactly identical as has been assumed above. Other more general theorems useful for design of multiple-loop feed-back amplifiers and servomechanisms may be derived from ..."

Courtesy of Mathematical Reviews R. Hamming, USA

970. **Hamaker, H. C., Taudin Chabot, J. J. M., and Willemze, F. G., The practical application of sampling inspection plans and tables,** *Philips tech. Rev.* **11,** 12, 362–368, June 1950.

For the practical execution of sampling inspection by unskilled factory personnel, the Philips Works in Holland have introduced a sampling table from which a suitable and efficient sampling plan can easily be derived for each particular case.

From authors' summary

980. **Stange, K., The formation law for the error formulas in the compensation of error measure series with the help of integer rational functions of growing order** (in German), *Z. angew. Math. Mech.* **29,** 7/8, 225–238, July/Aug. 1949.

A set of measured quantities . . . $x_{k-1}, x_k, x_{k+1}, \ldots$ corresponding to values of an argument at equal intervals of length τ can be smoothed by a formula (1) $\bar{x}_k = \sum_{v=-n}^{n} \lambda_v x_{k+v}$, where the λ's are determined from the best fitting (in the sense of least squares) polynomial of degree $R = 2r$, or $R = 2r + 1$. The author investigates the asymptotic behavior, as the n in (1) grows large, of what he calls the "physical error," namely, $f_P = (\sum_{v=-n}^{n} \lambda_v^2 \mu)^{1/2}$ and of what he calls the "mathematical error" f_M, where

$$f_M = \frac{x_k^{(2r+2)} \tau^{2r+2}}{(2r+2)!} \sum_{v=-n}^{n} v^{2r+2} \lambda_v.$$

To do this he expressed f_P and f_M by means of ratios of certain determinants. First- and second-order approximations are worked out and the accuracy of the second approximation is indicated by graphical comparison of the approximate with the true values. He also obtains formulas for the $\bar{x}$ expressed as functions of differences of even order and treats in considerable detail the special case $n = r + 1$, i.e., the case where two more points are used than there are coefficients in the polynomial.

Courtesy of Mathematical Reviews W. E. Milne, USA

981. **Dumas, M., and Maheu, P., Statistical methods and their applications in the field of industrial techniques. II** (in French), *Mémor. Artill. fr.* **24,** 2, 325–439, 1950.

A continuation of a series of articles by these two authors. Among topics treated are the uniform distribution, Poisson law, central limit theorem, the classification of distributions according to Karl Pearson, and the composition of two or more probability laws. A number of industrial examples are given. An interesting discussion is given of some of the difficulties underlying construction of sound inspection plans. Benjamin Epstein, USA

982. **Ortega, Miguel Garcia, Space structures** (in Spanish), *Rev. Cien. apli.* **4,** nos. 2, 3, 97–116, 209–226, Mar.-Apr., May-June 1950.

By using matrix notation, author establishes the relationships between displacements at ends of an element (bar) of a three-dimensional structure and forces (reactions) applied to them, in a comparatively simple and straightforward way. In the first article, the isolated bar is considered and force-displacement relations are established in the usual way, then expressed as matrixes referred to a coordinate system related to the bar in unstrained state. Equations of transformation to another arbitrary coordinate system are then given and the use of matrix calculus for performing such transformation is indicated.

Second article shows how equations of equilibrium for joints of the structure can be immediately written in matrix form. To actually solve the problem it is only necessary to translate these equations in scalar form and solve the resultant linear system.

The final setup becomes greatly simplified if symmetry exists in the geometry of the structure and the applied external loads. Means of attaining this simplification are also indicated.

Several forms for the systematic computation of the various steps are presented, and application of the method to a particular structure is worked out in detail. Ernesto Saleme, USA

983. **Bonnell, D. G. R., and Watson, A., An apparatus for measuring small changes in linear dimensions,** *J. sci. Instrum.* **27,** 1, 10–12, Jan. 1950.

In apparatus described, movement of the specimen is magnified by the change in the angle of reflection of a beam of light incident on a system of rotating mirrors. A device is incorporated to translate horizontal movements of the specimen into vertical movements on the extensometer. From authors' summary

984. **Cattelain, F., Measurement of linear variations of cables** (in French), *Ann. Trav. publics Belg.* **103,** 2, 261–266, Apr. 1950.

Two schemes—photogrammetric, and mechanical—used to measure elongations of prestressing cables of the Selayn bridge are described. Gage length of 12.17 m was marked off on cables and on Invar wire subjected to 20 kg tension. Photogrammetric method has been described in Bull. 17, *Société Belge de Photogrammetric*. In mechanical method, dials reading to 0.01 mm were used. Precision reported for either method is about 0.02 mm. Among advantages and disadvantages listed for both methods, author states systematic errors are eliminated through use of photogrammetric method. James P. Michalos, USA

985. **Mahl, Hans, Measurements of lengths and thicknesses in the electron microscope** (in German), *Arch. tech. Messen* **156,** T81–83, Aug. 1948.

986. **McDonald, Donald, Analog computers for servo problems,** *Rev. sci. Instrum.* **21,** 154–157, 1950.

The impedance equation connecting input and output voltage for an amplifier is specialized to correspond to (1) differentiation, (2) integration, and (3) summation. In the first two cases distortion in phase and magnitude occur, both depending on frequency. Under certain assumptions satisfied in practice, limits are found for these distortion terms in the case of an amplifier of gain 700, the design details of which are given. It is concluded that such low gain amplifiers are suitable for servo problem analog computers.

Courtesy of Mathematical Reviews R. Church, USA

987. **McCann, G. D., The California Institute of Technology electric analog computer,** *Math. Tables and other Aids to Computation* **3,** 501–513, 1949.

A brief description of the composition of the machine and details of some of its components. The kinds of problems treated and the general methods used are discussed. The bibliography lists a number of papers which contain more detailed information.

Courtesy of Mathematical Reviews S. H. Caldwell, USA

988. **van Wijngaarden, A., and Scheen, W. L., Table of Fresnel integrals,** *Proc. kon. Ned. Akad. Wet.* **19,** 4, 1–26 1949.

Five-decimal tables of Fresnel's integrals: $C(u) = \int_0^u \cos(\pi/2)t^2 dt$; $S(u) = \int_0^u \sin(\pi/2)t^2 dt$ for u ranging from 0 to 20 in .01

intervals are given, together with the modified second differences: $\delta^{2*} = \delta^2 - 0.184\,\delta^4$. An additional table contains values of the interpolation polynomials E_0^2 and E_1^2 for each 10th of the interval so as to facilitate use of Everett's interpolation formula. A short account of method followed in computation of tables and in checking values obtained, and numerical coefficients used in the series expansion are also presented. Ernesto Saleme, USA

989. Gusarova, R. S., On the boundedness of the solution of a linear differential equation with periodic coefficients (in Russian), *Prikl. Mat. Mekh.* **14**, 3, 313–314, May-June 1950.

Author proves by elementary means the following generalization of a theorem of Lyapunov. "If $p(x) \geqq n^2$, $-\infty < x < \infty$, $\pi \int_0^\pi p(x)dx \leqq 4(n+1) + n^2(\frac{1}{2}\pi^2 + 4)$, $n = 0, 1, 2, 3, 4$, then all solutions of $y'' + p(x)y = 0$ are bounded for $-\infty < x < \infty$." R. Bellman, USA

990. Erugin, N. P., Note on the integration in finite form of a system of two equations (in Russian), *Prikl. Mat. Mekh.* **14**, 3, p. 315, May-June 1950.

Author points out that the system of two real equations $dx/dt = u(x, y)$, $dy/dt = v(x, y)$ can be reduced to the complex equation $dz/dt = F(z)$, $z = x + iy$, if u and v satisfy the Cauchy-Riemann conditions, and illustrates this by means of the equation $dz/dt = az + bz^2$. This method may be quite useful in obtaining families of typical solutions. R. Bellman, USA

991. McLachlan, N. W., Ordinary nonlinear differential equations in engineering and physical sciences, Oxford, Clarenden Press, 1950, vi + 201 pp. $6.50.

Book is an attempt to give engineers and physicists a brief introduction to methods for treating ordinary nonlinear differential equations. Accordingly, since there exist no general methods but only numerous examples with recurring techniques, a restrictive selection has been made. Some specialists will object to the contents on the grounds that the examples of particular interest to them have been omitted; however, an objective evaluation shows that the excluded material, with one major exception, is more generally in the category of illustrative examples (although there is a bibliography of 211 references) rather than in the category of the methods. Methods included are: (1) Integration by inspection or simple transformation; (2) integration in terms of elliptic functions; (3) iteration; (4) perturbation; (5) expansion into a Fourier series and elimination of the secular terms; (6) derivation of equivalent linear equations; (7) application of Mathieu functions in approximating periodic solutions; and (8) isocline and other graphical and numerical procedures. Book is particularly strong whenever examples are taken from acoustics or vibration theory or the methods depend on Mathieu functions. The aeronautical engineer has been somewhat slighted in that Lanchester's phugoids, Hamel's solution for spiral flows, the equation of Blasius, and Pohlhausen's use of isoclines in boundary-layer theory (to mention only a few) are excluded. The major objection, however, is the omission of a discussion on use of analog and digital computers and the accompanying difficulties in satisfying boundary conditions, and this is surely the method most frequently used by engineers for solving other than obvious differential systems.

Book is well written and the typography and physical composition (no misprints were noted) are in harmony with the faultless tradition of the Oxford Press. All in all, the work will be valuable to anyone interested in making nonlinear analyses or in checking the reasonableness of the recorders of analog computers or the output tables of digital computers.

Knox Millsaps, USA

992. Théophile, Got, Determination of periodic stable solutions of certain quasi-harmonic differential equations (in French), *C. R. Acad. Sci. Paris* **230**, 612–614, 1950.

Poincaré showed that the system $\dot{x} = y$, $\dot{y} = -x + \mu P(x, y)$ has a limit-cycle for all small $\mu > 0$; here $P(x, y)$ is a polynomial, $P(-x, -y) = -P(x, y)$. Author introduces polar coordinates p, θ and obtains expression $p = p_0 + \sum_{n=1}^{\infty} \mu^n p_n(\theta)$ for a limit cycle; functions $p_n(\theta)$ have period π and are calculated by recursion. Similar expressions are obtained for the period and amplitude of a limit-cycle. Explicit results of these calculations are presented for the equation of van der Pol, in which $P(x, y) = (1 - x^2)y$.

Courtesy of Mathematical Reviews J. G. Wendel, USA

993. Murray, F. J., Linear equation solvers, *Quart. appl. Math.* **7**, 263–274, 1949.

This article considers automatic adjusting types of linear simultaneous equation solvers where the process is either continuous as in an analog-equation solver, or in discrete steps as in a digital computing routine. If $\epsilon_i = \sum_{j=1}^{n} a_{ij}x_j + b_i$, the only adjusting processes considered are assumed to be linear operators on the ϵ_i. "In the present article, we point out that if an adjusting type of machine is to operate successfully whenever the determinant A is not zero, then the square of the determinant must enter the indicial equation of the equations of motion for the machine. This necessary condition for successful operation rules out any linear feedback which does not involve using the a_{ij} twice. This result generalizes certain aspects of the necessity argument indicated in Goldberg and Brown" [*J. appl. Phys.* **19**, 339–345, 1948; AMR 1, Rev. 759].

Courtesy of Mathematical Reviews R. Hamming

994. Booth, A. D., An application of the method of steepest descents to the solution of systems of nonlinear simultaneous equations, *Quart. J. Mech. appl. Math.* **II**, part 4, 460–468, Dec. 1949.

Five procedures are outlined for "solving" systems of nonlinear simultaneous equations. Author asserts that his method requires only $1/n$ times the number of steps that are necessary in the Southwell and Synge methods; however, he admits that expressions used in his method are so complicated that high-speed electronic computing equipment is indicated. Reviewer believes that simplicity of the formulas far outweighs the requirements of extra iterations in methods to be used on such computing equipment. Harry D. Huskey, USA

995. Pipes, Louis A., The summation of Fourier series by operational methods, *J. appl. Phys.* **21**, 4, 298–303, Apr. 1950.

Author shows how the Laplace transform method may be applied to the summation of Fourier series of the type occurring in the theories of electrical and mechanical oscillations and conduction of heat. After a lucid exposition of the fundamentals of the method, author illustrates its use by considering four suitably chosen examples of some practical importance.

I. N. Sneddon, England

996. Marsilii, Paolo, A method for the approximate solution of problems of operational calculus (in Italian), *Elettrotecnica* 35, 258–263, 1948.

Given the function $F(p)$ in $F(p)/p = \int_0^\infty e^{-pt}f(t)dt$, the author tries to obtain an approximate evaluation of $f(t)$. In $f(t) = (2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty} e^{pt}p^{-1}F(p)dp$ he replaces p by $1/a$, and writes $f(t) = -(2\pi i)^{-1}\int_C e^{t/a}F(1/a)a^{-1}da$, where the line of integration is a circle C through the origin, with its center on the real axis. As all singularities of $F(p)$ are on the left of the straight line

$(c - i\infty, c + i\infty)$ there are no singularities of $F(1/a)$ inside or on C. Now the function $F(1/a)$ is approximated by a polynomial $F^*(1/a) = \alpha + \beta a + \gamma a^2 + \ldots + \nu a^n$. The coefficients $(\alpha, \ldots, \nu)$ are obtained by identification of $F(1/a)$ and $F^*(1/a)$ at $n + 1$ arbitrarily chosen points a_k on or inside C. The approximate value of $f(t)$ is

$$f^*(t) = -(2\pi i)^{-1} \int_C e^{t/a} F^*(1/a) a^{-1} da.$$

Some examples are considered. By graphical integration an upper bound for the absolute value of the error is found.

S. C. van Veen, Netherlands

Courtesy of Mathematical Reviews

997. **Rao, G. V. R., Upper and lower bounds on variational integral in compressible flow,** *J. aero. Sci.* **17,** 3, p. 182, Mar. 1950.

K. O. Friedrichs' method for formulating an adjoint minimum problem is applied to the variational integral for the linear perturbation potential of two-dimensional steady compressible flow. The integral can thus be bounded above and below. It is not clear to reviewer that as a consequence ". . . we can . . . gather information on the approximations involved in the velocities." Variational approximations can exhibit poor local behavior while still providing a good approximation for the variational integral.

Stephen H. Crandall, USA

998. **Stiefel, Edward, and Ziegler, Hans, Natural eigenvalue problems** (in German), *Z. angew. Math. Phys.* **1,** 2, 111–138, Mar. 1950.

Collatz ["Eigenwertprobleme"; AMR **3,** Rev. 604] has discussed in detail the differential equation approach to characteristic value problems; however, methods he developed do not apply to problems in which the characteristic values appear in the boundary conditions. Authors propose to overcome this difficulty and to facilitate the mathematical formulization of the problem by using the calculus of variations. Special problem of whirling of a cantilever shaft with end mass and axial thrust is used as the basis for a general discussion of two-dimensional vibration and stability problems. Correlation is obtained between theory of characteristic value problems based on the differential equation and theory founded on variational principles.

Robert P. Felgar, Jr., USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1023, 1030, 1236, 1418)

999. **Heck, O. S., Contribution to the analytic treatment of equilateral four-bar linkages** (in German), *Z. angew. Math. Mech.* 30, 8/9, 239–240, Aug.-Sept. 1950.

Paper shows how the inverse curve to a conic, referred to an arbitrary point, may be generated by an equilateral four-bar linkage. Relations between the parameters of the conic, coordinates of the reference point, and the lengths of the links permit determination of the conic from the linkage, or vice versa. It is claimed that this problem finds practical application in the generation of Piercy and Joukowsky profiles through link mechanisms.

G. A. Nothmann, USA

1000. **Whitehead, J. R., Surface deformation and friction of metals at light loads,** *Proc. roy. Soc. Lond. Ser. A., Math & Phys. Sci.* 201, no. 1064, 109–123, Mar. 1950.

A study has been made of the interaction of sliding metals when the normal load between them is very small. A new apparatus is described which enables friction to be measured, under controlled conditions of sliding, down to loads of a few milligrams. Deformation of the sliding surfaces has been studied by light and electron microscopy. These techniques have been used to investigate the validity of Amontons' law (proportionality of frictional force to load) over a very wide range of loads, from a few milligrams to several kilograms.

Experimental results on electrolytically polished copper reveal a departure from Amontons' law when the normal load is less than a few grams. A corresponding change is observed in the deformation within the track of sliding. At light loads friction is low and damage is slight. This is attributed to effect of the thin film of oxide formed by contact with atmosphere. A thicker film of oxide, formed by heating in air, reduces friction at heavy loads. Results of observations on copper surfaces prepared in various ways show that the influence of surface roughness upon friction is not great.

Results of further experiments show that friction of silver on silver, and of aluminum on aluminum is constant and Amontons' law holds over the whole range of loads. It appears that the thin film of oxide on aluminum is penetrated even at the lightest loads. Results are discussed in relation to those for copper, and the difference in behavior of the various metals is attributed to the contrasting properties of their oxides.

Isolated experiments on sapphire and diamond show that the coefficient of friction is low and constant for these nonmetals, and a few experiments made on boundary lubricated metals are recorded.

In spite of the deviations from Amontons' law on some metals, the results of all the experiments on dry sliding emphasize the essential similarity of the frictional process and of the basic mechanism over an enormous range of loads.

From author's summary by E. I. Shobert II, USA

1001. **Morris, J., The whirling of a spinning top,** *Aero. Quart.* **2,** part I, 9–14, May 1950.

The whirling phenomenon in rotating shaft systems has again become a problem of considerable importance, primarily in the recent development of lightweight prime movers with revolving rotors at high speeds, and also in connection with the mechanism for operating contra-propeller systems. The whirling of a top in consequence of an unbalance is treated here. The ordinary symmetrical top may have only one whirling speed and this will be above the minimum speed for stability in the vertical position. The unsymmetrical top however, in certain circumstances, may have two whirling speeds between which the unforced disturbed motion is exponentially unstable. Problem considered by author is that of a top spinning on a perfectly rough horizontal plane.

Th. Pöschl, Germany

1002. **Honnell, Pierre M., The absolute calibration of a Brush accelerometer and its response to square waves of displacement and velocity,** *Trans. Amer. geophys. Un.* **31,** 5, part I, 690–695, Oct. 1950.

Paper presents a simple absolute dynamic calibration technique employing the gravity acceleration as a standard against which the imposed dynamic acceleration is directly compared. Method is suitable for vibration pickups or shake tables. Illustrated by calibration of Brush pickup. Response of pickup to various wave shapes is shown.

Martin D. Schwartz, USA

1003. **Mitchell, D. B., Tests on dynamic response of cam-follower systems,** *Mech. Engng.* **72,** 6, 467–471, June 1950.

Paper reports a preliminary experimental study of three cam profiles: parabolic, harmonic, and cycloidal. Records of accelera-

tion-time and velocity-time are presented for a range of cam angular velocities. It is concluded that for the cycloidal profile, peak forces on the follower are "generally slightly lower than those produced by other profiles." Also, forces corresponding to follower vibration are "much smaller" for the cycloidal cam. It is emphasized that in all cases, construction of follower is a controlling factor in the reduction of oscillatory force, and that follower should be stiff with low mass. R. E. Roberson, USA

1004. **Storchi, E., Integration of indeterminate equations of statics of continuous systems on a surface of revolution** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* **7**, 5, 227–231, Nov. 1949.

This paper is concerned with the general solution of Cauchy's statical equations for a continuous medium. For two-dimensional, three-dimensional, and four-dimensional spaces of zero or constant curvature the problem was solved by B. Finzi [title source, (6) **19**, 578–584, 620–623, 1934]. The author solves it for the case of a surface of revolution embedded in a three-dimensional flat space. In general the stresses are given as linear combinations of the first, second, and third partial derivatives of a single stress function. The known special cases of the plane (Airy) and the sphere (Finzi)—and surfaces applicable to them—are exceptional in that they are the only ones in which first and second derivatives of the stress function suffice. The author indicates that for a general two-dimensional surface it can be expected that the analogous result will involve fourth derivatives also. C. Truesdell, USA

1005. **Schaffner, Hans, Problems and methods of nonlinear mechanics** (in German), *Bull. Assn. Suisse Elect.* **41**, 17, 629–633, Aug. 1950.

Expository paper. Stephen H. Crandall, USA

Gyroscopics, Governors, Servos

(See also Rev. 1215)

1006. **Ulanov, G. M., On the maximum deviation of the regulated quantity in a transient process** (in Russian), *Avtomat. i Telemekh.* **9**, 168–175, 1948.

The mathematical problem is that of determining the maximum of $x(t)$ where x satisfies the linear operational equation $f(p)x = y$ ($p = d/dt$), and we are given that $|y| \leq a$. Using standard Laplace transform method of solution, author discusses a number of special cases, giving bounds for x in terms of a and $f(p)$.

R. Bellman, USA

1007. **Oppelt, W., The setting diagram for regulators** (in German), *Z. angew. Math. Mech.* **30**, 5/6, 188–189, May-June 1950.

Linear equations for governed system with self-governing capability and for governor with relay mechanism including the dashpot are represented by vector diagram, taking into account the lag of the system, e.g., as in the governing of heat. From this diagram, drawn for case of the limit of stability, a new equation is drawn. From it a diagram is set up showing relation between characteristic data at the limit of stability. Diagram gives information quickly as to the characteristic data which are needed to obtain stable governing. M. Nechleba, Czechoslovakia

1008. **Ryabov, B. A., Auto-oscillations in some servosystems conditioned by the presence of dry (Coulomb) friction** (in Russian), *Doklady Akad. Nauk SSSR* **73**, 2, 283–286, July 1950.

Theoretical analysis of a viscously damped system of n generalized coordinates, with the addition of Coulomb damping in only one of the coordinates. Substantially standard procedures are set up for solution of $n - 1$ homogeneous differential equations and one nonhomogeneous equation containing the dry friction term. Author shows that the period of self-excited oscillations in such automatic control systems depends only on the constants of the follow-up system and the oscillations may be eliminated by appropriate adjustment of their values. Walter W. Soroka, USA

1009. **Stout, Thomas M., A note on control area,** *J. appl. Phys.* **21**, 11, 1129–1131, Nov. 1950.

Time integral of error for step input has been suggested as a measure of control-system performance. Paper develops correctly a method for performing this integration, provided system-transfer function can be expressed as ratio of two polynomials. The polynomials are factored into products of first-order binomials with time independent time constants. Time integral of error is shown to be the sum of time constants in denominator minus the sum of time constants in numerator.

Reviewer comments that the open-loop transfer function is frequently known as a product of simple factors, but that the finding of simple factors for the closed loop is difficult. For case of finite control area, denominator factors into a free integrator, $1/s$, times several binomials. Control area in this case is the reciprocal of the loop gain. Minimization of control area calls for high (infinite) gain. Similar situation exists for systems with steady-state error. Hence, usefulness of minimum control-area criterion is questioned.

Paper will encourage further search for control criteria, but is of no use to control designers at present.

John C. Sanders, USA

Vibrations, Balancing

(See also Revs. 998, 1001, 1005, 1009, 1096, 1118, 1121, 1136, 1149, 1172, 1230, 1307)

1010. **Thomson, William Tyrrell, Mechanical vibrations,** New York, Prentice-Hall, Inc., 1949, viii + 222 pp. $5.

An introductory textbook of free and forced vibration of basic systems. Treatment of one-degree-of-freedom and two-degree-of-freedom systems is followed by consideration of more advanced problems of torsional systems and beams by methods of Holzer, Rayleigh, and Stodola. A short chapter is devoted to elastic bodies where simple analytical solutions exist. The important class of self-excited vibration is omitted. Text and illustrations are clear; examples are worked out to illustrate the text; and problems for the student are given at the end of each chapter. Reviewer believes this is an excellent textbook for classroom instruction. A. C. Hagg, USA

1011. **Strelkov, S. P., Introduction to the theory of vibrations (Vvedenie v teoriyu kolebanii)** (in Russian), Moscow-Leningrad, Gosud. Izdat. Tekhn.-Teor. Lit., 1950, 344 pp.

This text is addressed to the engineer rather than to the mathematician. Mechanical and electrical systems are considered. While most of the treatment is for linear systems with constant coefficients, nonlinear systems, parametric excitation, and distributed parameters are also discussed. N. Levinson, USA

1012. **Krzywoblocki, M. Z., Vibration in composite beams,** *Aircr. Engng.* **22**, 258, p. 223, Aug. 1950.

Using Rayleigh-Ritz method, a procedure is outlined for determination of the natural frequency of two elastic beams, A and B, connected together at the middle of beam A, with additional

concentrated weights elastically supported along A. Both the uncoupled flexural and torsional cases are discussed. Procedure is applicable to the free vibration of airplane wing and fuselage, with engines, bombs, etc., as suspended weights.

William T. Thomson, USA

1013. **Ayre, R. S., and Jacobsen, L. S., Transverse vibration of a two-span beam under the action of a moving alternating force,** *J. appl. Mech.* **17, 3, 283–290, Sept. 1950.**

Paper contains theoretical and experimental studies. Because of the many assumptions, particularly that of zero damping, primary value of theoretical portion is to show that, as a result of movement of the force along the beam, (a) there exist at least two distinct exciting frequencies at which peak responses of each natural mode will occur; (b) these particular exciting frequencies usually differ noticeably from the natural frequencies of the beam. Experimental results are given which confirm the theory in these respects but, as expected, show stresses appreciably lower than those predicted.

B. Smilg, USA

1014. **Woinowsky-Krieger, S., The effect of an axial force on the vibration of hinged bars,** *J. appl. Mech.* **17, 1, 35–36, Mar. 1950.**

The nonlinear problem of free vibrations of a bar with hinged ends is reduced to a nonlinear ordinary differential equation which is solved with the aid of elliptic functions. A table gives numerical values of frequencies versus amplitude of vibration. Procedure is analogous to one employed by Kirchhoff for the corresponding problem of the vibrating string.

E. Reissner, USA

1015. **Wenk, Edward, Jr., A theoretical and experimental investigation of a dynamically loaded ring with radial elastic support,** *David W. Taylor Mod. Basin Rep.* 704, NS 870–001, 71 pp., July 1950.

Author investigates free vibrations of circular rings on radial elastic supports and subjected to radial impulse in plane of ring. Depth of ring cross section is small compared to diameter. Rotary inertia and shearing distortions are considered negligible and have been ignored in the equations of motion. The theoretical treatment makes use of familiar product solution with model shapes given in Fourier series.

An elaborate experimental investigation is described validating theoretical conclusions to a high degree of accuracy.

R. M. Rosenberg, USA

1016. **Druszewski, Edwin T., and Houbolt, John C., The calculation of modes and frequencies of a modified structure from those of the unmodified structure,** *Nat. adv. Comm. Aero. tech. Note* 2132, 23 pp., July 1950.

Paper deals with the modes and frequencies of a structure to which lumped masses and/or concentrated spring forces or torques have been added. It is assumed that the modes and frequencies of the unmodified structure are known. Technique is based on Galerkin's method, modes of unmodified structure being used as the arbitrary functions. Process used in finding the characteristic equation is parallel to that for deriving the orthogonality conditions. It contains as unknowns the generalized coordinates, the frequency, and the concentrated loads and torques, which are in turn functions of the coordinates and frequency. A novel feature is the elimination of the coordinates and use of a set of equations in which the loads and torques are the unknowns. Each term of the equations is an infinite series, but authors show that good accuracy is attained by use of only a few terms of the series. Examples cover case of concentrated masses and springs added to a uniform beam. The mathematical development is too brief for nonspecialists in vibrations, and the use of equation (8), to determine the modes, is obscure in the examples. Method should be useful in flutter calculations on aircraft wings.

Bernard Etkin, Canada

1017. **Cambi, Enzo, Theory for the design of a metal tower against seismic vibrations** (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* **6, 608–616, May 1949.**

A theory is worked out for computing the response to earthquakes of transmission-line towers such as those proposed for the high-tension cable across the Straits of Messina. Tower is treated as an elastic beam of variable section rigidly clamped at its base and carrying a concentrated mass M at the top. The variation in section along beam is approximated by piecing the beam together out of several portions with a constant mass per unit length in each portion and a variation in moment of inertia proportional to $(1 - \gamma x)^2$, where γ is a constant and x is distance from base. A solution is obtained in terms of Bessel functions after satisfying conditions of continuity at junctions of the successive beam portions and end conditions at base and at top carrying the concentrated mass. A closed solution is given for a fixed-free beam with $M = 0$, in which the moment of inertia varies as $(a - bx)^2$ throughout height of beam. The natural modes of vibration of the tower are finally used to express deflection of tower in response to a given external seismic acceleration.

W. Ramberg, USA

1018. **Soroka, Walter W., Free periodic motions of an undamped two-degree-of-freedom oscillatory system with nonlinear unsymmetrical elasticity,** *J. appl. Mech.* **17, 2, 185–190, June 1950.**

System under consideration is a two-degree-of-freedom torsional system, the shaft being clamped at one end, and carrying two disks; the restoring moment from first shaft is linear, that from second shaft varies according to a curve composed of two straight lines in an unsymmetrical manner. Author treats problem by making use of linear equations in their respective fields and fitting the results together. Author gives samples of motions in seven diagrams, and points out that these diagrams show "unconventional forms" and have, with increasing amplitudes, "an unexpected trend [to a] growth of the number of oscillations of the linear shaft in a period [of the second shaft]."

Reviewer considers results shown are not as strange as they may appear at first glance. Thinking in terms of an "equivalent spring constant" for second shaft, which will decrease from high to low values as the amplitudes increase, one might interpret the given results in a qualitative way quite readily: With decrease of the average spring constant there will be not only a change of frequencies (of the two modes), but also a change of the amplitude ratios. The ratio $\kappa = A_1/A_2$ for the first mode becomes very low, that for the second mode goes toward high values. The angular coordinates θ_1 and θ_2 of both disks will have contributions from both modes; but in θ_2 the lower mode will prevail (because of the high value of κ_{I}) whereas in θ_1 the higher mode will prevail (because of the low value of κ_{II}). This agrees with the evidence.

K. Klotter, Germany

1019. **Sponder, E., On the representation of stability region in vibrations by means of the Hurwitz-determinants** (in German), *Schweiz. Arch.* **16, 3, 93–96, Mar. 1950.**

Given a frequency equation of nth degree where the coefficients depend on parameters x and y. Paper shows stability region in x, y plane to be between curves $D_{n-1}\,O$ and $a_n\,O$, where D_{n-1} is the

$n-1$th Hurwitz determinant and a_n is the constant term in frequency equation. N. O. Myklestad, USA

1020. Benz, Walter, Bending vibrations of shafts with single rotors (in German), *Motortech. Z.* **11**, 3, 68–73, May-June 1950.

Relations between whirling and operating velocities of a narrow rotor on a flexible shaft are derived in terms of moment of inertia of rotor and elastic constants of shaft. The complete range of parameter interrelation is discussed, for equal as well as for opposite directions of whirl and rotation.

Unbalance, interaction between two rotor systems, or gas and inertia forces of the system may give rise to critical speeds. The last possibility is studied in detail on the basis of experimental results (single cylinder engine: oscillographs of shaft deflection for the complete speed range and optical measurements of deflection configurations) and by a thorough dynamic force analysis of a reciprocating engine, in which many orders of critical speeds are of interest. For such engines, the critical speeds for which whirl and rotation are of opposite direction are claimed to be of particular importance. G. A. Nothmann, USA

1021. Spaetgens, T. W., Holzer method for forced-damped torsional vibrations, *J. appl. Mech.* **17**, 1, 59–66, Mar. 1950.

Shows generalized Holzer table for Diesel-engine forced vibrations at a prescribed frequency under influence of gas torques and damping, both between masses and at masses. The tables are in complex numbers, containing two columns for each one in the usual case. Applied to an actual engine installation carrying a fluid damper. J. P. Den Hartog, USA

1022. Fraeys de Veubeke, B. M., Crankshaft-propeller vibration modes as influenced by the torsional flexibility of the engine suspension, *J. aero. Sci.* **17**, 5, 288–296, May 1950.

An investigation is made of crankshaft-propeller torsional oscillations coupled with the torsional oscillations of the engine as a whole. The method used is an extension of Biot's [*J. aero. Sci.*, Jan. and July 1940]. The equations ofoscillation of any crankpin and of the crankcase of a long crankshaft engine are first set up. Expressions for dynamic moduli (essentially the ratio of torque amplitudes to angular amplitudes), which are here functions of frequency, are then given corresponding to various types of propeller gear reductors. The equations are shown to lead in general to a transcendental frequency-determinant equation, and a practical method of solving this equation is discussed. The results of a numerical example are shown in which the engine suspension is assumed as rigid, "hard," and "soft" (low resonant frequency), respectively. M. Morduchow, USA

1023. Malkin, I. G., On the theory of oscillations of quasilinear systems with many degrees of freedom (in Russian), *Prikl. Mat. Mekh.* **14**, 353–370, July-Aug. 1950.

In the present paper, author continues to follow the program which he seems to have drawn for himself—to extend to more or less general nonanalytical systems of differential equations results already acquired for analytical systems. He now considers a quasilinear system of order n

$$dx_s/dt = \Sigma a_{sr}x_r + \mu f_s(t,x_1, \ldots, x_n),\ s = 1, 2, \ldots, n$$

where the a_{sr} are constants, μ is a small parameter, and the functions f_s are continuous with period 2π in t and have Fourier expansions. Relative to x_s they have continuous first partial derivatives satisfying a Lipschitz condition in a certain domain. The problem is to find periodic solutions of period 2π which for $\mu = 0$ tend to similar solutions of the limiting linear system. In a previous paper [*Prikl. Mat. Mekh.*, **13**, 1949], author discussed what happens when linear system has a periodic solution depending upon a certain number of parameters. Results of that paper are applied here with particular emphasis on resonance, i.e., when the matrix of the a_{ij} has a certain number of characteristic roots which are zero, or of the form $\pm mi$, m an integer. The solutions are obtained by successive approximations giving rise to series which are shown to converge [Further relevant references: Malkin, I. G., op. cit., **14**, 1950; A. I. Lurie, op. cit., **12**, 1948, fasc. 4]. S. Lefschetz, USA

1024. Ayre, Robert S., Jacobsen, Lydik S., and Phillips, Ar is Steady forced vibration of a non-conservative system with variable mass; a pumping system, *J. Franklin Inst.* **250**, 4, 315–337, Oct. 1950.

A system described by a generalized nonhomogeneous Floquet equation $\ddot{x} + \varphi(t)x = \psi(t)$, where $\varphi(t)$ is a stepwise changing mass function and $\psi(t)$ is a piecewise continuous forcing function, is analyzed theoretically and experimentally. Assumption that system behaves as two single-degree-of-freedom systems in two linear mass ranges, use of initial conditions and displacement continuity conditions lead to solutions in four different classes of motion. Classes are characterized parametrically by static deflection, forcing amplitude, forcing frequency, and the two natural frequencies.

Example treated is that of deep-well reciprocating pump with negligible damping and sufficient external power to make the forcing function independent of the load. Rotational analog of translational pump system is used as experimental aid for computation. Results include: (1) Single resonance for this compound system with two natural frequencies occurs between the two frequencies. (2) Displacement is linear function of forcing amplitude if ratio of static deflection to forcing amplitude is constant. (3) Over-travel of the secondary inertia occurs in two of the four classes of motion.

Experimental and theoretical results agree closely except near resonance. H. G. Cohen, USA

1025. Putnam, C. R., An oscillation criterion involving a minimum principle, *Duke math. J.* **16**, 633–636, 1949.

Let $q(x)$ be real and continuous on $0 \leqq x < \infty$. Let $y(x)$ be real and continuous and $y'(x)$ piecewise continuous for $a \leqq x < \infty$, where $a \geqq 0$, and let $y(a) = 0$. Let $\int_a^\infty y^2 dx = 1$ and let $\int_a^\infty (y'^2 + |q|y^2)dx < \infty$. Let $\mu(a)$ denote the greatest lower bound of $\int_a^\infty (y'^2 + qy^2)dx$ for the class of $y(x)$ described above. Clearly $\mu(a)$ is nondecreasing. Then if and only if $\mu(a) < 0$ for all $a \geqq 0$, the differential equation $y'' - qy = 0$ is oscillatory on $0 \leqq x < \infty$, that is, every solution has an infinite number of zeros. N. Levinson, USA

1026. Braunbek, Werner, Natural oscillations of finite system of mass points (in German), *Ann. Phys. Ser.* **6**, 4, 3/4, 169–173, Nov. 1948.

1027. McCann, G. D., and MacNeal, R. H., Beam-vibration analysis with the electric-analog computer, *J. appl. Mech.* **17**, 1, 13–26, Mar. 1950.

Authors have greatly expanded the scope of this technique by developing analogies involving circuit elements independent of frequency, and capable of handling transient as well as static and steady-state problems. Coupling between degrees of freedom, combined beams, "lumped constant" elements, and other factors may be considered at the same time. The differential equations for beams having several separate as well as coupled degrees of freedom are replaced in paper by finite difference equations repre-

senting equivalent lumped mechanical systems of the order of 10 cells, and equivalent electrical circuits are indicated. Both mass-inductance and mass-capacitance analogies are employed.

Certain circuit requirements for practical application, and for minimizing errors, are discussed. Accuracy of method, as carried out on the CalTech computor, was first investigated by checking the solution for bending modes of a uniform cantilever beam against the analytical one. Resonant frequencies were obtained within 0.2%, and steady-state mode shapes through the fifth accurately determined using a limited number of cells. Response of a cantilever beam subjected to an impulsive motion at its base was then obtained and checked against the nearest available analytical solution. Here, a repeated step current excited the network, and response as a function of time at any desired point along the beam was presented on an oscilloscope screen.

It is pointed out that nonuniformity of beam section, damping, rotary inertia, and other conditions difficult to consider by numerical methods may readily be taken into account. One practical example, a tapered-wing analysis subjected to coupled bending and torsion, is worked out for symmetric (zero slope) and for unsymmetric (pinned) constraint at the fixed end. Bending as well as induced coupled torsional mode shapes are plotted. Effect of coupled torsion upon the uncoupled bending solution was found appreciable in the second symmetric mode. Initial problem as well as computer set-up time were sufficiently short to render generalized studies very practical. C. W. Gadd, USA

1028. **Mathewson, Alice W., Calculation of the normal vertical flexural modes of hull vibrations by the digital process,** *David W. Taylor Mod. Basin*, 706, NS 712-065, Feb. 1950.

Calculation of natural frequencies of ship vibration has, until the present, been a discouraging process. Setting up an equivalent system is long, the calculations are laborious and require a high degree of accuracy, and results are usually disappointing. Even if some experimental values of hull frequency are available, the mode shape is almost never available and results usually disagree with calculated values. Now appears a ship whose natural frequencies and mode shapes have been accurately determined.

At the same time, a method of calculation is presented adopting the old Holzer scheme, as applied to beams by Prohl and Myklestad, to use of automatic computers. This reduces labor, errors, and time involved in computing the natural frequencies and mode shapes from the mass, elastic characteristics of the ship. Calculated results agree surprisingly well with experimental values for as high as fifth mode of vertical vibration, with the greatest error 13% low for lowest frequency, and remainder all within 7%. Usual experience for carefully calculated ships is to obtain a comparable accuracy for first mode with much poorer agreement in higher modes, even when effects of shear and rotary inertia are included.

By considering this work in connection with other work, reviewer believes that author is fortunate in the ship which was chosen. Although calculation method is much simpler than others, it is not basically more accurate, nor is there any reason to believe that the mass, elastic characteristics of the ship are more accurately known. Reviewer believes that differences in experimental and calculated values of natural frequencies of most ships can be traced to neglecting effect of local resonances of decks, superstructures, or, in some cases, whole sections of ship's structure. In this ship it appears that such resonances either did not occur or, if they did, they were located at a mode or possessed a frequency almost identical with that of one of calculated modes. It is heartening to see one case work out so beautifully and to have at last a method developed which makes calculations practical.
F. E. Reed, USA

Wave Motion, Impact

(See also Revs. 1335, 1381)

1029. **Sauter, Fritz, The problem of the semi-infinite fluid with a mechanical disturbance on its surface (two-dimensional problem)** (in German), *Z. angew. Math. Mech.* **30,** 5/6, 149–153, May/June 1950.

As a preliminary study of the propagation of small vibrations through a semi-infinite elastic body, author considers propagation of sound through a semi-infinite fluid when prescribed pressure disturbances are applied to the surface. Mathematically, the problem is that of solving the $2 + 1$ dimensional wave equation with the somewhat unusual boundary conditions that the wave function $\Phi(x, z, t)$ be zero initially and that it approach zero at large distances from the disturbance. Furthermore, the derivative $\partial\Phi/\partial t$ is prescribed as a function of t and x on the surface of the fluid $z = 0$.

Author solves the problem using an appropriate influence function and verifies the solution. He also treats explicitly the case where the pressure on the surface is a point disturbance.
H. E. Moses, USA

1030. **Schmidt, W., Study of the reflection axiom for the oblique impact of elastic spheres** (in German), *Z. angew. Math. Mech.* **30,** 5/6, 182–184, May/June 1950.

Relations between velocities and directions of travel before and after oblique impact of a sphere on a flat wall are derived. Analysis introduces a tangential coefficient of restitution, analogous to the corresponding coefficient for normal impact. In general, rotation after impact is induced and the angles of incidence and reflection are unequal. Analysis is extended to oblique impact for initial rotation combined with translation, and to impact between two spheres. Tentative conclusions regarding astral impact are advanced. G. A. Nothmann, USA

1031. **Trilling, Leon, The impact of a body on a water surface at an arbitrary angle,** *J. appl. Phys.* **21,** 2, 161–170, Feb. 1950.

Problem described in title is treated with following restrictions: Viscosity, surface tension, compressibility, and gravity are neglected; free-surface condition is linearized to become $\Phi = 0$ on still-water surface, where Φ is velocity potential (in particular, effect of splash is neglected); body is assumed to move with constant velocity and no rotation. Because of the linear boundary conditions, the potential may be expressed as a sum of two potentials arising from horizontal and vertical components of velocity of body. Following shapes are treated exactly, where in each case body is started in motion impulsively when half submerged: infinitely long elliptic cylinder, sphere, ellipsoid of revolution, and general ellipsoid. Author discusses application of results to problem of finding approximately the drag history as a body enters water up to the point when an entrance cavity begins to form. Application is made to case of a sphere entering at a 45-deg angle. General approach follows closely that of Sedoff (*Trudy Tsent. Aero-Gidrodinam. Inst.* no. 187, 1934); however, Sedoff worked out, as specific examples, only two-dimensional cases.
J. V. Wehausen, USA

1032. **Merten, Kenneth F., Rodríguez, José L., and Beck, Edgar B., A comparison of theoretical and experimental wing bending moments during seaplane landings,** *Nat. adv. Comm. Aero. tech. Note* 2063, 36 pp., Apr. 1950.

A smooth-water-landing investigation was conducted with a small seaplane to obtain experimental wing-bending-moment time histories together with time histories of the various parameters necessary for the prediction of wing bending moments during

hydrodynamic impact. Experimental results were compared with calculated results which include inertia-load effects and effects of air-load variation during impact. Responses of the fundamental mode were calculated with the use of the measured hydrodynamic forcing functions. From these responses, the wing bending moments due to the hydrodynamic load were calculated according to the procedure given in R. & M. no. 2221. Time histories of the experimental and calculated wing bending moments showed good agreement both in phase relationship of the oscillations and in numerical values. Effects of structural flexibility on wing bending moment were large, dynamic component of the total moment being as much as 97% of the static component. Changes in the wing bending moment due to variation in air load during impact were of about the same magnitude as the static-load component.

From authors' summary by E. G. Stout, USA

1033. Miles, John W., On certain integral equations in diffraction theory, *J. Math. Phys.* **28,** 223–226, 1950.

The problem of the diffraction of a plane wave by a ribbon or a slit of infinite length in an infinite screen is formulated as an integral equation which may be solved by Mathieu functions. Some identities involving Mathieu functions are derived from these integral equations. A. E. Heins, USA

Elasticity Theory

(See also Revs. 986, 1029, 1054, 1061, 1068, 1070, 1073, 1077, 1087, 1150, 1153, 1186)

1034. Rothman, M., Isolated force problems in two-dimensional elasticity, *Quart. J. Mech. appl. Math.* **3,** part 3, 279–296, Sept. 1950.

Solutions are given for the stresses and displacements in an infinite plate with circular, elliptic and nearly polygonal holes loaded by one or more concentrated forces at edge of hole. Method of analysis is due to A. C. Stevenson [*Proc. roy. Soc. Lond. Ser. A*, **184,** p. 129, 1945] and is one in which stresses and displacements are expressed in terms of two complex potentials. Author notes agreement of some of his results with certain solutions previously obtained by other methods. Numerical values for the tangential stresses at the edge of elliptic and nearly square holes are given for special loadings. John E. Duberg, USA

1035. Ghizzetti, Aldo, On the states of plane tension of an elastic body (in Italian), *Ann. Mat. pura appl.* (4) **29,** 125–130, 1949.

Author obtains a new and simple reduction of the problem of plane elastic stress: $t_{iz} = 0$, $i = 1, 2, 3$, and t_{ij} independent of z for $i, j = 1, 2, 3$. By use of integrability conditions he shows that it is necessary and sufficient for a solution that the extraneous force be solenoidal and independent of z and that

$$u = \alpha z^2 - \alpha x^2 + 2\sigma^{-1}\beta xy - \sigma^{-1}\alpha y^2 + \sigma^{-1}(1-\sigma)\gamma x + 2E^{-1}(1+\sigma)\partial\Phi/\partial y - \phi,$$

$$v = \beta z^2 - \sigma^{-1}\beta x^2 + 2\sigma^{-1}\alpha xy - \beta y^2 + \sigma^{-1}(1-\sigma)\gamma y - 2E^{-1}(1+\sigma)\partial\Phi/\partial x + \psi,$$

$$w = -2z(\alpha x + \beta y + \gamma),$$

$$T_{xx} = \sigma^{-1}E(2\beta y + \gamma) + 2\partial^2\Phi/\partial x\partial y - (1+\sigma)^{-1}E\partial\phi/\partial x,$$

$$t_{yy} = \sigma^{-1}E(2\alpha x + \gamma) - 2\partial^2\Phi/\partial x\partial y + (1+\sigma)^{-1}E\partial\phi/\partial x,$$

$$t_{xy} = \partial^2\Phi/\partial x^2 - \partial^2\Phi/\partial y^2 - (1+\sigma)^{-1}E\partial\phi/\partial y,$$

where E is Young's modulus, σ is Poisson's ratio, α, β, and γ are arbitrary constants, Φ is a function determined by the force field alone, and $\phi + i\psi$ is an analytic function of $x + iy$. These results constitute a notable simplification over the usual treatment in that instead of Airy's biharmonic stress function only an ordinary analytic function $\phi + i\psi$ is required. The author indicates a class of boundary problems which are reducible to the Neumann problem and another reducible to the Dirichlet problem.

C. Truesdell, USA

1036. Sengupta, A. M., Stresses in some aeolotropic and isotropic circular disks of varying thickness rotating about the central axis, *Bull. Calcutta math. Soc.* **41,** 129–139, 1949.

Stress distribution and deformation in a rotating cylindrically aeolotropic thin plate are determined. The thickness variations considered are of the form $r^{-\lambda}$ and $r^{\alpha}\exp(-\lambda r^{\beta})$, where r is the radial coordinate. Usual assumptions of thin plate theory are used. Author also reports the stress system associated with a rotating isotropic disk whose thickness variation is of the form $\exp(-\lambda r^2)$. G. F. Carrier, USA

1037. Green, A. E., and Shield, R. T., Finite elastic deformation of incompressible isotropic bodies, *Proc. roy. Soc. Lond. Ser. A*, **202,** 1070, 407–419, Aug. 1950.

Following three problems in the theory of deformation of incompressible highly elastic materials, which are isotropic in their undeformed state, are solved for a general form of the strain-energy function: (1) Torsion of a right-circular cylinder; (2) deformation, due to centrifugal force, of a right-circular cylinder rotated about its axis; and (3) inflation of a thick spherical shell. Problem (1) has previously been solved by Rivlin and results of present paper agree with those obtained by him.

Authors derive the stress-strain relations in tensor form and then specialize them to cylindrical or spherical polar coordinates as required.

In problem (3), the result is specialized to case of a thin spherical shell and it is shown that, when strain-energy function W for the material is given in terms of the principal extension ratios λ_1, λ_2, and λ_3 by $W = C(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3)$, where C is a constant, the inflating pressure at first rises to a maximum as shell is inflated and then falls with further inflation. This is in qualitative agreement with results which are observed if the thin shell is made of vulcanized rubber. R. S. Rivlin, England

1038. Jung, H., A contribution on Love's displacement function (in German), *Ingen.-Arch.* **18,** 3, 178–190, 1950.

Exact solutions of the following problems are given: (1) Half space with prescribed normal pressures; (2) half space with prescribed normal displacement for $r \leq a$, free conditions for $r > a$; (3) thick infinite plate arbitrarily loaded (a special case is also treated); (4) infinite cylinder with radial loading; (5) semi-infinite cylinder with arbitrary normal displacement at end; (6) Hertz's problem of the rigid sphere on an elastic half space.

Rotational symmetry is assumed in each case. Integrals of particular solutions with respect to a parameter are fitted to boundary conditions by Hankel and Fourier transforms. Numerous references to German literature are given.

Sneddon [*Proc. Camb. phil. Soc.* **44,** 492–507, 1948; AMR 2, Rev. 711] has given a more nearly complete solution of (2).

Roy C. T. Smith, Australia

1039. Blokh, V. I., Stress-functions in the theory of elasticity (in Russian), *Prikl. Mat. Mekh.* **14,** 415–422, July-Aug. 1950.

General solution of stresses and strains using stress functions are surveyed. Fundamental equations are expressed in vectorial form. Author shows the development of these equations for a

system of orthogonal coordinates. Conditions of compatibility of the primary tensor are given with reference to previous papers of the author, also stresses and strains. Boundary conditions are expressed generally in a vectorial form.

Z. Bažant, Czechoslovakia

1040. **Mitrinovitch, Dragoslav S., Relation between an unsolved problem of the theory of elasticity and a problem solved by Darboux and Drach** (in French), *C. R. Acad. Sci. Paris* **231**, 5, 327–328, July 1950.

P. Neményi and the reviewer [*Proc. nat. Acad. Sci.* **29**, 159–162, 1943; *Trans. Amer. Math. Soc.* **58**, 96–166, 1945, § 19] reduced the general equilibrium problem of the classical membrane theory of thin elastic shells to that of solution of the single differential equation $f''/f = A^2p$, where A is an arbitrary constant and p is a given function determined by the form of the shell. The author shows that this problem in turn is equivalent to that of solving $\eta''/\eta = \phi + A^2$, where ϕ is another given function. While the reviewer's inverse method of solution yielded simple exact solutions for numerous infinite families of shell forms, it was haphazard. The author states that the results concerning $\eta''/\eta = \phi + A^2$ previously obtained by Darboux, Drach, and himself enable one systematically to construct infinite sequences of shell forms for which the equilibrium problem can be solved by quadratures.

C. Truesdell, USA

1041. **Berman, M. E., Concerning the center of flexure** (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **72**, 27–30, 1950.

By straightforward derivation author expresses the coordinates of center of flexure as line integrals of the harmonic function of torsion. Contrary to author's claim, method is known [cf., for example, Sokolnikoff and Specht, "Mathematical theory of elasticity," New York-London, McGraw-Hill, 1946]; however, his final expressions seem to be somewhat easier to compute.

Courtesy of Mathematical Reviews H. I. Ansoff, USA

1042. **Rasmussen, Bent Højlund, The complete solution of the Clapeyron equation** (in Danish), *Dansk Selsk. Bygningsst. Bygnings. Medd.* **21**, 4, 117–127, 1950.

Article deals with computation of the reciprocal matrix of a system of Clapeyron equations. From author's summary

1043. **Sen, Bibhutibhusan, Note in the deformation produced by some symmetrical distribution of variable loads on the plane boundary of a semi-infinite elastic solid,** *Bull. Calcutta math. Soc.* 41, 77–82, 1949.

A simple general method is given for solving the problem of a semi-infinite isotropic elastic solid under action of a load distributed uniformly over any area on its plane boundary. The general method may be applied to a nonsymmetrical load distribution, but author only gives the details for a symmetrical distribution over a circular area, which is such that load varies as a function $f(r)$ of distance r from center of circle. The particular paraboloidal distribution for which $f(r) = 1 - r^2/a^2$ over a circular area of radius a is given in full detail, while results for two load distributions varying exponentially with r have been stated briefly.

Courtesy of Mathematical Reviews R. M. Morris, USA

1044. **Hay, G. E., Beam under concentrated loading,** *Proc. Symp. appl. Math.* **3**, 13–26, 1950.

Plane strain problem of a region bounded by two confocal ellipses on which two equal and opposite forces act at end points of the major axis of the cross section is solved. After indicating the method of curvilinear coordinates and complications involved, solution is achieved by conformally mapping the region onto a region bounded by two concentric circles and then using method of complex variables developed by N. I. Muskhelishvili. Recurrence relations are given for the coefficients in the series expansion of the stress function. These equations are such that no simple method of successively solving for the unknowns appears to be available, so that author offers a perturbation method to obtain the desired coefficients. In general, author states, a proof of convergence of the series representing the stress function seems difficult to attain; however, numerical calculations for a special case are said to indicate convergence. No numerical results are given.

Marvin Stippes, USA

1045. **Oldroyd, J. G., Finite strains in an anisotropic elastic continuum,** *Proc. roy. Soc. Ser. A, Math. Phys. Ser.* **202**, 1070, 345–358, Aug. 1950.

Using same notations author extends discussion of previous paper [AMR **4**, Rev. 212] on invariance properties required of the equations of state of a homogeneous continuum, by considering thermodynamic restrictions on the form of the equations for an elastic solid deformed from an unstressed equilibrium configuration. Finite strain-stress-temperature relations are expressed in terms of a free-energy function without assuming isotropy. The equations of state are derived by considering quasistatic changes in an elastic solid continuum; results then apply to non-ideally elastic solids in equilibrium, or subjected to quasistatic changes only, and to ideally elastic solids in general motion. If a material element of a solid is subjected to an arbitrary finite constant deformation at constant temperature, the solid is called *ideally elastic* if the final stresses are attained instantaneously, and *non-ideally elastic* if the final stresses are attained only after a finite or infinite time.

A necessary and sufficient compatibility condition for finite strains at different points of a continuum is derived. Torsion of an anisotropic cylinder is briefly discussed using a free-energy function which is a generalization of Mooney's prototype for rubber. From author's summary by A. E. Green, England

1046. **Sengupta, A.M., Thermal stresses in isotropic circular disks of varying thickness rotating about a central axis,** *Bull. Calcutta math. Soc.* **41**, 199–207, 1949.

An elastic disk rotating about its axis and subjected to a radial thermal gradient is considered. The stress distribution in such situations is found for cases (1) thickness proportional to r^λ, temperature $= t_0 + kr^n$; and (2) thickness proportional to $\exp(\lambda r^q)$ and temperature proportional to $\log(b/r)$. The parameter r is the radial coordinate; all others are constants.

G. F. Carrier, USA

1047. **Green, A. E., and Zerna, W., The equilibrium of thin elastic shells,** *Quart. J. Mech. appl. Math.* **3**, part I, 9–22, Mar. 1950.

Authors derive the differential equations for the deformation of an arbitrary isotropic elastic shell of constant thickness by means of tensor methods, using general coordinates on the middle surface. Basis of theory is the equilibrium conditions for tractive forces and moments. Use of a displacement vector obviates the need for compatibility equations. Only linear terms in the derivatives of the displacement vector are retained in the strain tensor. Terms of the order of ratio of thickness to minimum radius of curvature are neglected in comparison to unity. Shearing stresses on plane elements parallel to middle surface are assumed to vary parabolically throughout thickness.

Authors feel that their approximations are less restrictive than

those of other investigators. To some extent, this is true, although problems of large deflections and problems involving small radii of curvature lie outside the scope of their theory. Results of the theory would have greater practical value if final equations were expressed in scalar form, and if physical significance of each term were clearly illustrated, so that one could discern more readily which terms may be neglected in special problems. H. L. Langhaar, USA

Experimental Stress Analysis

(See also Revs. 984, 985, 1002, 1105, 1156, 1163, 1164, 1173, 1174)

1048. Fisher, W. A. P., Measuring fringe orders in photoelasticity, *Research* **3,** 7, 298–303, July 1950.

Available techniques for quantitative measurement of relative retardation are reviewed with special emphasis on the Tardy method. Relative retardation being proportional to principal stress difference, photoelasticians will find useful this straightforward explanation of the art of "reading between the lines" of an isochromatic diagram. L. E. Goodman, USA

1049. Jessop, H. T., and Wells, M. K., The determination of the principal stress differences at a point in a three dimensional photoelastic model, *Brit. J. appl. Phys.* **1,** 7, 184–189, July 1950.

A method is described of finding the directions of the principal axes of stress at any point of a thin slice cut from a "frozen stress" model. Using a polarizing microscope and a universal tilting stage, slice is oriented until one of the principal stress axes coincides with an axis of the stage as indicated by maintained extinction of the light for subsequent rotation about this axis. Two principal stress directions are found experimentally and the third graphically. Directions of the principal stress axes are recorded using stereographic projections. It is then a simple matter to set each principal plane normal to polarized light for measurement of the principal stress differences, using a compensator in microscope tube. At least two slices will be needed for complete information since it is undesirable to tilt the slice more than 45°.

M. M. Leven, USA

1050. Hall, E. O., An optical method for studying the deformation of mild steel, *Proc. phys. Soc.* **63,** part 9, 369 (B), 724–726, Sept. 1950.

Small spheres of tin are mounted at the middle of the gage length of thin wire specimens of mild steel. The image of a light source is reflected by the spheres on a photographic plate. If strain is uniform, sphere moves with half the speed of the moving head of the testing machine. If strain is localized between sphere and moving head, sphere does not move. If strain is localized between sphere and fixed head, sphere moves with same speed as moving head. By using several spheres along the length of the specimen a complete history of localized flow is obtained. This equipment has been used to study Lüders' bands in normal and strain-aged steels. E. A. Davis, USA

1051. Pollett, W. F. O., and Cross, A. H., A continuous-shear rheometer for measuring total stress in rubber-like materials, *J. sci. Instrum.* **28,** 8, 209–212, Aug. 1950.

Authors have described a plastimeter of Weissenberg type with added improvements. The instrument has some advantages of measurements over earlier types but is not suitable for all rubber or rubberlike mixes. Several kinds of mixes of rubber were used as samples to measure the normal and tangential stresses over various temperature ranges. One curve is drawn of the relationship of temperature dependence to tangential and normal stress for a grade 7 polythene. Temperature range was 120 to approximately 200 C.

Reviewer believes the instrument described has limited research value in stress measurements of samples of rubber or rubberlike materials. Walter T. Daniels, USA

1052. Campus, F., Mechanical instruments to measure deformations (in French), *Centre belge Rech. navales,* 11 pp., 1950.

Article discusses mechanical extensometers and strain gages for use in shipbuilding and work of a similar nature. Author collaborated with R. Dantinne during and after World War II in building improved extensometers of the dial type with spherical contact points. Several of these are described and illustrated.

W. C. Johnson, Jr., USA

1053. Schaal, Alfred, Experimental determination of the effective depth of penetration in x-ray stress determination (in German), *Z. Metallk.* **41,** 9, 293–295, Sept. 1950.

In order to properly measure strains in metals by means of x-rays, it is necessary to know their depth of penetration, particularly when the surface differs much from the inside of the metal. A theoretical estimate of this penetration is not always satisfactory. Author describes experiments which lead to an estimate of depth of penetration of x-rays by comparing the intensities of x-ray diffraction originating in the metal and in a thin metallic film, which covers the surface. The thickness of the film at which the diffraction from the underlying metal is not more observable indicates the effective depth of penetration of x-rays. Results are tabulated for penetration of cobalt radiation into iron, aluminum, and copper. Romon Smoluchowski, USA

1054. Sáenz, A. W., Determination of residual stresses of quenching origin in solid and concentric hollow cylinders from interferometric observations, *J. appl. Phys.* **21,** 10, 962–965, Oct. 1950.

Presents a theoretical procedure for obtaining residual stresses in transparent cylinders from interferometric observations. Assumptions are a stress system which is radially symmetrical and independent of axial and angular coordinate variations, and principal stress directions which coincide with radial, tangential, and axial directions. Derivation is based on the Maxwell-Neuman stress-optic law. Results are said to be of interest for the analysis of stress in quenched glass cylinders.

W. O. Richmond, Canada

1055. Kolsky, H., Stress-birefringence in polystyrene, *Nature* **166,** 4214, 235–236, Aug. 1950.

Tests conducted on compression specimens of polystyrene, "Transpex II," at 16 C show that the birefringence is initially positive and proportional to the applied stress for instantaneous loading, giving a stress optical coefficient of about 310 psi per fringe per inch. Author states that if stresses above 1500 psi are maintained, the birefringence decreases with time, and if the stress is then removed a negative birefringence results. For stresses above 11,000 psi the birefringence passes through zero and becomes negative even when the stress is maintained.

E. O. Stitz, USA

1056. Perls, Thomas A., A retraction-type displacement gage, *David W. Taylor Mod. Basin Rep.* 651, NS 724-004d, 28 pp., Mar. 1950.

Report covers design, construction, and testing of a retraction-type displacement gage, which consists of a resistance wire rigidly attached to the moving structure and held tight by a device such as a steel spring or a rubber band. The wire passes through a

special fixed mercury contact, and the change in resistance between the moving structure and the contact is detected and recorded on suitable instruments. Information relative to theoretical considerations, properties of rubber bands, recording devices, mercury contacts, experiments, results, and applications may be found in the main text.

Gage is simple, light, and rugged and suitable for displacements greater than 0.1 in. A motion will be followed by the gage if the velocity and acceleration do not exceed 200 ft/sec and 1000 g. Higher accelerations may be obtained by modification. Displacements up to 15 in. have been measured. The gage should be very useful in a variety of experiments where it is necessary to measure displacements. Shop drawings for the mercury contact are presented. George A. Hawkins, USA

1057. **Hrennikoff, A., Framework method and its technique for solving plane stress problems,** *Publ. int. Assn. Bridge Struct. Engng.* 9, 217–248, Nov. 1949.

Paper deals with application of the methods of finite differences to plane stress problems. It is a continuation of a paper by same author in *J. appl. Mech.*, Dec. 1941. R. M. Wengren, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1013, 1022, 1041, 1044, 1146)

1058. **Mutermilch, J., Application of the finite-differences calculus to calculations based on the exact equation of the curve** Same source as Rev. 1113 et seq. in this issue), 183–193.

When the deflections of the bars are great, the equation $Ely'' = M$ fails and, on the other hand, integration of the exact equation sometimes requires the use of the theory of elliptical functions. Author suggests substitution of a finite-differences equation for the exact equation of the curve and adapts Runge's method. Convergence of the obtained successive values for deflections is slow and, therefore, author had to work out a method to make the convergence quicker. W. Wierzbicki, Poland

1059. **Sakharov, I. E., Bending of a rod under centrifugal force** (in Russian), *Prikl. Mat. Mekh.* **13,** 329–330, 1949.

Consider a homogeneous horizontal rod of contant cross section, rotating with constant angular velocity about a vertical axis to which one of the ends of the rod is clamped. If r is measured perpendicular to the axis, and $w(r)$ is the (small) deflection, then $Bw^{iv} - [T(r)w']' = f(r)$, where B is the constant flexural rigidity, $T(r)$ is the tension, and $f(r)$ is the transverse load, including the weight of the rod. The end conditions are $w(0) = w'(0) = w''(L) = w'''(L) = 0$, where L is the length of the rod. Using $w'''(L) = 0$, a second-order nonhomogeneous ordinary differential equation is obtained for w', which is solved explicitly in terms of confluent hypergeometric functions [reference is made to C. Wells and R. Spence [*J. Math. Physics* **24,** 51–64, 1945]. The deflection $w(r)$ is then obtained by integration, and all three remaining arbitrary constants are determined to fulfill the other three end conditions.

Courtesy of Mathematical Reviews J. B. Diaz, USA

1060. **Swida, W., Calculation of a closed circular ring supported statically indeterminately and arbitrarily loaded normal to its plane** (in German), *Ingen.-Arch.* **18,** 4, 242–249, 1950.

Closed ring of rectangular or circular cross section, supported in many points so as to allow only angular deviation in the vertical tangential plane and arbitrarily loaded normal to its plane, is analyzed generally by a system of three-moment equations, derived by method of least work, analogously to a continuous beam. Coefficients of the moments at supports in three-moment equations are functions of central angles between the radii to the supports; their values for any cases of central angles are given in a table, also the coefficients for a concentrated load acting in several points. As a special case, a horizontal circular segment fixed at both ends is treated. Z. Bažant, Czechoslovakia

1061. **Reissner, H. J., and Wennagel, G. J., Torsion of noncylindrical shafts of circular cross section,** *J. appl. Mech.* **17,** 3, 275–282, Sept. 1950.

The problem of torsion of noncylindrical shafts is reduced to solution of the differential equation

$$\partial(r\partial\omega/\partial r)/\partial r + 2\partial\omega/\partial r + r\partial^2\omega/\partial z^2 = 0$$

with the boundary condition to be satisfied along the generatrix $z(r)$: $dz/dr = (\partial\omega/\partial z)/(\partial\omega/\partial r)$. (Here r radial, z axial cylindrical coordinates; $\omega(r,z)$ angular displacement.) Using Saint Venant's semi-inverse method, author prescribes the solutions of the differential equation and adapts the form of the generatrix to the boundary condition. As a special case of a general class of integrals [mentioned by A. E. H. Love, "Theory of elasticity," London, 1927, sect. 226A] the following solutions of the differential equation are investigated numerically:

$$\omega(r,z) = A \exp(qz)\cdot r^{-1}J_1(qr);\quad \omega(r_1z) = B \exp(qz)\cdot r^{-1}N_1(qr);$$
$$\omega(rz) = C \sin(qz)\cdot r^{-1}J_1(iqr)$$

(A, B, C, q are constants; J_1, N_1 are Bessel functions). Shape of the generatrix, shear stresses, and displacements can be easily expressed by Bessel functions. Solutions lead to some hollow and full cross sections of technical interest.

H. Kauderer, Germany

1062. **Pode, Leonard, A method of determining optimum lengths of towing cables,** *David W. Taylor Mod. Basin Rep.* **717,** 14 pp., Apr. 1950.

1063. **Broughton, D. C., Clark, M. E., and Corten, H. T., Tests and theory of elastic stresses in curved beams having I- and T-sections,** *Proc. Soc. exp. Stress Anal.* **8,** 1, 143–156, 1950.

An experimental strain analysis of curved beams with various I- and T-sections disclosed that the flanges had a tendency to act independent of the beam as a whole. For such wide-flanged beams there also occurs transverse bending of the flanges as a secondary effect. In order to explain the first of these discrepancies from the classic theory of Winkler-Bach, authors propose a new theory, based on assumptions: (a) The component of stress due to pure bending is given by the classic Winkler-Bach formula, e.g., assuming plane sections of beam to remain plane in bending; (b) component of stress due to direct load is not uniformly distributed over cross section, but is a maximum at inside edge of beam, according to a certain stress concentration factor K; finally, (c) authors define K as being the ratio of maximum bending stress, occurring in a pure bending of a curved beam having a given cross section, to maximum bending stress in a straight beam with same cross section and being exposed to same bending moment. Reviewer is not able to follow the logic of authors' theory, as disclosed by above assumptions (a), (b), (c), but would rather explain the independent bending action of the flanges with shear deformation of the web.

F. Odqvist, Sweden

1064. **Archibald, Frank R., Torsional stress in close-coiled helical springs,** *Engineering* **169,** 4398, 541, May 1950.

Article is a discussion of Dr. Morley's paper on torsional stress

in close-coiled helical springs and points out that it is reasonable to assume that the transversal shear stress, caused by the load on the spring, is also higher toward center of coil, while the other investigators have assumed it to be the same as in a straight bar. This assumption would increase the curvature correction factor, as given by the Wahl formula, with from 0.46% for springs of an index of 3 to 1.5% for springs of an index of 10. The controversial point raised by author is of mainly academic interest since the referred-to increase of the factor is not material.

O. R. Wikander, USA

1065. Blair, J. S., Stresses in tubes due to internal pressure, *Engineering* **170,** 4416, 218–221, Sept. 1950.

The large number of formulas available for determining the strength of tubes under internal pressure indicates a lack of agreement on the proper theory of failure to be used in combining the longitudinal, hoop, and radial stresses. Treatment given here is novel, both as regards the suggested formula and the extension of the strain-energy theory involved. A test by author, using combined bending and internal pressure, indicated that Haigh's theory (strain-energy) gives better results than the others. For simple working formulas, Barlow's (outside diameter) formula is sufficiently accurate for elastic conditions and the mean-diameter formula for plastic conditions. Marshall Holt, USA

1066. Guest, James J., Stresses in tubes due to internal pressure, *Engineering* **170,** 4426, 412–413, Nov. 1950.

This short article, a letter to the editor, discusses Blair's paper of same title (see preceding review). It points out that careful tests on gun steels has led to acceptance of Mises-Hencky criterion of failure. The conclusion by Dr. Blair that the Haigh theory is the true law for yield contradicts earlier findings and implies that a reconsideration of the problem is necessary.

The Guest law is not equivalent to the shear-stress criterion since a certain volumetric stress was included in the formulation of the law and the shear-stress criterion was suggested as a compromise for ductile metals.

Many references are given. Marshall Holt, USA

1067. Feely, F. J., Jr., and Goryl, W. M., Stress studies on piping expansion bellows, *J. appl. Mech.* **17,** 2, 135–141, June 1950.

Paper deals primarily with thermal expansion joints which are constructed by crimping thin stainless-steel disks into shallow cones which are resistance-welded together at their inner and outer edges. This type is convenient for high temperature and low pressure piping.

Authors use elementary beam theory to derive a formula which gives total stress induced in the material as a result of combined effects of pressure and movement. Validity of the approximations used has been verified by laboratory strain-gage measurements. A relationship between several variables in design of the disks has also been determined and serves as a basis for dimensioning them to achieve the most economical proportions. When this relationship is satisfied, maximum permissible movement per disk can be obtained.

The problem of designing another type of expansion joint for high-pressure service is discussed briefly, and some preliminary laboratory data are presented on a commercial joint of this type.

From authors' summary by Ch. Massonnet, Belgium

1068. Mitra, D. N., On the flexure problem of a limaçon and some other boundaries, *Bull. Calcutta math. Soc.* **41,** 153–158, 1949.

Asserting that Stevenson's result for the center of flexure of a limaçon [*Proc. London math. Soc.* (2) **45,** 126–143, 1939] is incorrect, author derives another result which does not contain the known result for a cardioid by a factor 2. However, author asserts that Stevenson's result is in error by use of an unnecessary factor 2 arising early in Stevenson's paper. [Reviewer has not verified these errors.] As a second example, the flexure problem for a cylinder whose cross section is the inverse of an ellipse is treated and is said to agree with known results. D. L. Holl, USA

1069. Ashwell, D. G., The anticlastic curvature of rectangular beams and plates, *J. roy. aero. Soc.* 708–715, Nov. 1950.

Paper discusses a refinement of the usual theory for bending of thin plates, which becomes important when principal radius of curvature of plate is small. With the increasing use of thin wall construction this should prove a useful contribution. No more complicated mathematics is used in analysis than is required in ordinary simple beam theory, and it is shown that many of the formidable expressions appearing in the solutions may be simplified by a slight approximation.

Author indicates it is permissible to neglect the transverse slope of the beam in deriving the equation of deflected shape, as is usually done, but it is not clear whether he intends to make a similar approximation for principal radius of curvature of the beam, although this seems most likely.

Frank A. Blakey, Australia

1070. Mitra, D. N., On flexure functions of a semi-circular cylinder, *Bull. Calcutta math. Soc.* **41,** 125–128, 1949.

Author employs function theoretic methods introduced by S. Ghosh [same Bull. **39,** 1–14, 1947; AMR **1,** Rev. 1312]. These canonical flexure functions are determined for a semi-circular cylinder according to the method employed in previous papers [same Bull. **40,** 173–182, 1948] and are said to agree with those of Ghosh in an earlier paper [same Bull. **40,** 77–82, 1948; AMR **2,** Rev. 19]. D. L. Holl, USA

Plates, Disks, Shells, Membranes

(See also Revs. 1015, 1036, 1037, 1046, 1069, 1135)

1071. Chang, Fo-Van, Trigonometric series applied to the bending of long rectangular plates to a cylindrical surface, *J. Franklin Inst.* **249,** 4, 279–286, Apr. 1950.

The known problem of cylindrical bending of a uniformly loaded rectangular plate [see Timoshenko, S., "Theory of plates and shells," 1940, p. 4] is solved using trigonometric series and energy methods. In reviewer's opinion, the same form of solution could be obtained more easily and directly from the differential equation for displacement and, in any case, has no advantages over the known solution. A plate on an elastic foundation is also considered, using same method. A. E. Green, England

1072. Favre, Henry, and Chabloz, Eric, Study of bent circular plates with linearly variable thickness (in French), *Z. angew. Math. Phys.* **1,** 5, 317–332, Sept. 1950.

The thickness h of a nonuniform circular plate of radius a is taken as a linear function of r in the form

$$h = [1 + \lambda(r/2a - 1)]h_0,$$

λ being a suitable parameter and h_0 the thickness at $r = \frac{1}{2}a$. Bending of such plates subjected to uniform pressure and with either clamped or simply supported edges is discussed by method of successive approximations. Each approximation yields a linear differential equation with homogeneous coefficients, whose solution can be obtained in a finite number of terms. Numerical re-

sults are graphically represented for the displacement of the middle surface and the bending couples. It is found that λ should lie between 0.2 and 0.4 for a clamped plate and -0.4 and -0.2 for a simply supported plate. In existing treatments of such problems h is taken to be proportional to r so that $\lambda = 1$. The problem may be treated by superposing the solution for $\lambda = 1$ on that for uniform thickness. B. R. Seth, India

1073. **Hasse, H. R., The bending of a uniformly loaded clamped plate in the form of a circular sector,** *Quart. J. Mech. appl. Math.* **3,** part 3, 271–278, Sept. 1950.

The solution for deflection is expressed in the form

$$w = W + \sum_{n=0}^{\infty} A_n v_n$$

where W is a particular solution satisfying the differential equation and the v's are biharmonic functions. W, $\nabla^2 W$, and v_n are required to be zero on the boundary. The A's are determined to satisfy clamped condition of plate. For the given restrictions on W and v_n, author shows that clamped conditions lead to an infinite set of simultaneous equations represented by the following

$$\int\int [v_n q/D + \nabla^2 v_n \sum_{s=0}^{\infty} A_s \nabla^2 v_s] dS = 0$$

where the integration is over the area S of the plate.

Case of a semicircular plate is considered in detail. Except for the corners, good accuracy is obtained by using only first six terms of the series. Application to the analogous problems of plane stress and plane strain is indicated. Gerald Pickett, USA

1074. **Thorne, C. J., Symmetrically loaded rectangular plates fixed at points,** *Bull. Univ. Utah* **39,** 10, 9 pp. + tables, Nov. 1949.

Deflection functions are given as a sum of biharmonic polynomials with coefficients determined by the slopes and deflections of equally spaced points on each edge of the plate. Numerical coefficients of point load and uniform load for zero slope and deflection of equally spaced points on each edge, full fixed, not full fixed are given. Design data are calculated and plotted for eleven examples along the center lines, diagonals, and the edges.

Paul M. Naghdi, USA

1075. **Guzman, Arturo M., and Luisoni, Cesar J., New solutions for some particular loadings of plane plates** (in Spanish), *Univ. Nac. La Plata Publ. Fac. Ci. Fisicomat,* no. 196, serie segunda, 18, **4,** 268–287, 1949.

Galerkin's approximate method for finding the deflection of a thin plate subject to a normal distributed load q (i.e., for finding a real-valued function $w(x, y)$ satisfying the partial differential equation

$$\partial^4 w/\partial x^4 + 2\partial x^4 w/\partial x^2 \partial y^2 + \partial^4 w/\partial y^4 = \Delta\Delta w = q/N$$

in a plane domain D, where q is a given real-valued function defined on D, N is a real constant, and w is subject to certain homogeneous boundary conditions), starting with n functions $\varphi_1, \ldots, \varphi_n$ which satisfy the homogeneous boundary conditions, consists in determining n real numbers $a_1, \ldots, a_n$ from the system of linear equations

$$\sum_{i=1}^{n} a_i \, {}_D\!\int\int \varphi_j \Delta\Delta\varphi_i \, dx\,dy = {}_D\!\int\int N^{-1} q \varphi_j \, dx\,dy, \qquad [*]$$

$j = 1, \ldots, n$, and then taking the function $\sum_{i=1}^{n} a_i \varphi_i$ as an approximation to the deflection w. The authors refer to [*] as Galerkin's orthogonality condition. By an argument involving the principle of virtual displacements the authors replace [*] by another condition which may be applied when both concentrated and distributed loads occur. Using their condition they treat numerically the wholly simply supported and the wholly clamped rectangular plate with a concentrated load at the center, computing both displacements and moments, and compare their results with those of Timoshenko ["Theory of plates and shells," New York-London, McGraw-Hill, 1940], Marcus ["Die Theorie Elastischer Gewebe . . .," 2nd ed., Berlin, Springer, 1932], and Stiles [*J. appl. Mech.* **14,** A-55–A-62, 1947; AMR **1,** Rev. 62]. The simply supported rectangular plate with partial rectangular load (i.e., q a nonzero constant on a smaller concentric rectangle and zero outside it) is also dealt with numerically. In each case it is stated that the present method, from the standpoint of numerical work, is to be preferred to the previous ones.

Courtesy of Mathematical Reviews J. B. Diaz, USA

1076. **Sengupta, H. M., On the bending of an elastic plate, I,** *Bull. Calcutta math. Soc.* **41,** 163–172, 1949.

Author solves problem of a thin elliptic plate with a clamped edge, bent by a concentrated load acting at any point on its surface. No numerical results are given. H. D. Conway, USA

1077. **Southwell, R. V., On the analogues relating flexure and extension of flat plates,** *Quart. J. Mech. appl. Math.* **3,** part 3, 257–270, Sept. 1950.

The displacement of a flat plate bent by transverse loading and the extensional (Airy) stress-function in "plane strain" or in "plane stress" are governed by equations of identical form; and the boundary conditions have identical form when edge displacements are specified in the flexural, edge tractions in the extensional problem; so, mathematically, in these circumstances, only a single problem is presented. This, the first "analog" relating flexure and extension, is well known.

A "second analog," relating the flexural problem when edge-tractions with the extensional problem when edge-displacements are specified, is believed to have been first propounded in 1941 [L. Fox and R. V. Southwell, *Phil. Trans. roy. Soc. C,* **1,** p. 15, 1941; *A,* **239,** p. 419, 1945]. By introducing two quantities U and V, analogous with the components u and v of extensional displacement, it permits a treatment of the flexural problem by any method—e.g., "two-diagram (relaxational) technique"—which yields extensional solutions of this second type.

In this paper, both analogs are combined in an inclusive statement covering the perforated (multiply connected) plates which were discussed in 1948 [R. V. Southwell, *Proc. roy. Soc. Lond. Ser. A,* **193,** p. 147, 1948; AMR 2, Rev. 169]. Reasons are stated for believing that two-diagram technique is preferable in problems governed by mixed boundary conditions.

From author's summary by H. G. Hopkins, England

1078. **Meriam, J. L., Differential analyzer solution for the stresses in a rotating bell-shaped shell,** *J. Franklin Inst.* **250,** 2, 115–133, Aug. 1950.

Theory treats stresses caused by centrifugal forces in axially symmetric rotating elastic shell of constant thickness whose meridians are arcs of circles tangent to axis of rotation. Since circumferential displacements are zero, there are two unknowns in the problem; e.g., normal and meridional displacements. Author adopts certain functions of displacements and shear on circumferential sections of the shell as dependent variables. Differential equations are then separable. By virtue of axial symmetry, ordinary differential equations are obtained. Small deflection theory linearizes the equations. Solutions of the equations have been obtained with the aid of a differential analyzer, by a method which is discussed in considerable detail.

H. L. Langhaar, USA

1079. Aquaro, Giovanni, On the calculation of the deformations of an elastic spherical layer (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **7**, 289–297, 1949.

A complete system of particular solutions of the classical equations of equilibrium in terms of displacements is established. It is then applied to the solution of the second boundary-value problem (given surface tractions) for the spherical shell. The coefficients of the resulting series are obtained by solving systems of linear algebraic equations of order six.

Courtesy of Mathematical Reviews W. Leutert, USA

1080. Broglio, L., Introduction of a general theory of shells of translation (in French), *Inter. Assn. Bridge Struct. Engng. Fin. Rep.*, 3rd Congr., 553–564, Sept. 1948.

General theory of shells of translation, which provides a complete solution of the problem. By means of a change of variable quantities and by introduction of a stress function, problem is reduced to study of a rectangular membrane stretched and fixed along its boundary. Without any particular difficulty, method enables one to study shells that are nonsymmetrical and nonsymmetrically loaded, for which author stresses necessity of adding to equilibrium conditions (which are the only ones used) a condition of compatibility expressed by Menabrea's theorem.

Method provides a solution in case of concentrated loads also. Solution of a shell with parabolic directrixes, subjected to a load evenly distributed on horizontal projection in proposed method, is reduced to the known one of a rectangular membrane fixed along its boundary, uniformly stretched, and evenly loaded.

From author's summary

1081. Haas, A., The calculation of reinforced concrete roof shells, I, II (in Dutch), *Ingenieur* **62**, 12, 36; Bt 9–14, 43–48; Mar. 24, Sept. 8, 1950.

The fundamentals of the theory are briefly set forth. A valuable survey of literature on the subject is inserted.

W. L. Esmeijer, Holland

1082. Micks, W. R., Minimum weight of stiffened cylindrical shells in pure bending, *J. aero. Sci.* **17**, 4, 211–216, Apr. 1950.

Author shows that total weight (expressed by ratio $A/M^{2/3}$) depends on a structural index M/D^3 and a frame spacing ratio L/D, where A is average volume of panel and frame per unit length of cylinder, M bending moment, D diameter of shell, and L frame spacing.

Previous test data determine an envelope curve for the average allowable compressive stress in a panel f_c, and also determine dimensionless coefficient above which panel-type failure of shell would occur. This information, combined with a shape factor and modulus of elasticity for the frame, gives a simple expression for ratio $A/M^{2/3}$ which permits construction of design curves showing f_c, L/D and M/D^3. Optimum values of these quantities corresponding to minimum weight can be found. The expression permits separate curves to be constructed for panel material and frame material to show relative amounts of weight.

This application of principles of design for minimum weight was reported by Shanley [title source, **16**, March, Oct. 1949; AMR **4**, Rev. 151]. Secondary effects are omitted and analysis does not consider instabilities induced by load conditions other than pure bending. Herman A. Lang, USA

1083. Tolotti, Carlo, Statics of inextensible and elastically flexible surfaces (in Italian), *G. Mat. Battaglini* (4), **2** (78), 128–150, 1949.

In previous papers [*Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* (8) **1**, 187–192, 369–374, 605–609, 1946], author has generalized Beltrami's differential equations for the deformation of flexible but inextensible membranes by introducing terms arising from flexural elasticity. He now formulates the principle of virtual work in a form applicable to a deformed surface. By using the Gauss-Codazzi equations he deduces his former differential equations and associated boundary conditions as well.

C. Truesdell, USA

1084. Wyman, Max, Deflections of an infinite plate, *Canad. J. Res.* **28**, sec. F. 3, 293–302, May 1950.

A solution is given of the problem of a loaded, infinite, elastic plate supported on an elastic foundation. Problem relates to transporting equipment over frozen lakes, to landing aircraft on frozen lakes, and also to roadway design. Specialization is made to the case of a uniform distribution of load over a circular area, and simple expressions involving Kelvin's functions (ber x, bei x, ker x, and kei x) are found for the maximum displacements and stresses. Previously published solutions have given these quantities in terms of infinite series. Results found here agree with those obtained by the reviewer and R. P. N. Jones [*R.A.E. Report* no. S.M.E. 3397, 1947.] H. G. Hopkins, USA

1085. Woinowsky-Krieger, S., Calculation of an infinite plate elastically supported by a semi-space (in German), *Ingen.-Arch.* **17**, 1 & 2, 142–148, 1949.

Author points out the advantages of treating some problems in soil mechanics and allied subjects by using the analogy of an infinite elastic plate resting on a semi-infinite elastic solid. Employing the usual assumptions of the thin plate theory and the results of his previous paper [title source, **3**, 236–250, 1932] he obtains numerical values for deflection, pressure, and stress when the plate is acted upon by (1) a point load, (2) a circular line load. A comparison is also made with results given by the Hertz theory.

B. R. Seth, India

1086. Sturm, R. G., O'Brien, H. L., Wetterstrom, E., and Evans, J., Stresses in "head-to-shell" juncture of pressure vessels, *Weld. Res. Suppl.* **15**, 6, 285–292, June 1950.

A résumé of literature, theories in the design of pressure vessel head, application of these theories, tests of stress distribution under pressure, and correlation of test results with theory.

From authors' summary

1087. Read, W. T., Jr., Effect of stress-free edges in plane shear of a flat body, *J. appl. Mech.* **17**, 4, 349–352, Dec. 1950.

Author determines approximately the tangential stiffness of a rectangular shear block subjected to a uniform relative tangential displacement of two parallel faces, remaining faces being free from tractions. This stiffness is defined as the shear force on the upper and lower face required to produce a unit relative displacement. Upper and lower bounds for stiffness, in terms of elastic constants and thickness ratio, are obtained by means of the Prager-Synge energy method [*Quart. appl. Math.* **5**, 1947]. The width of the block is considered large compared to its length, and the problem is treated as one of plane strain.

E. Sternberg, USA

1088. O'Chou, Chang, Two dimensional theory of stiffened plates, *Aero. Quart.* **2**, part I, 1–8, May 1950.

Approximate equations are established for the stretching of plates reinforced by stringers and ribs, and equations are solved for two elementary problems. Solutions of these problems, and of many others, are already well known in the theory of ortho-

tropic plates where the governing equations are similar to those given in paper. A. E. Green, England

1089. **Johansen, K. W., On integration of the differential equation for thin shells without bending,** *Inter. Assn. Bridge Struct. Engng. Fin. Rep.*, 3rd Congr. 597–600, Sept. 1948.

Buckling Problems

(See also Revs. 1058, 1060, 1116, 1117, 1123, 1125, 1136, 1144)

1090. **Ashwell, D. G., The Euler buckling load of a strut with crossed pins,** *Engineering* **169,** 4404, p. 709, June 1950.

A derivation is given for the buckling load of a column loaded at its ends by pin joints, having pins at right angles to column axis but not parallel with each other. Numerical results are presented for a column having a cross section such that the moments of inertia about all axes passing through its centroid are equal. For a column with pins making an angle of 45° with each other, the buckling load is about 30% higher than if the pins were parallel with each other. Method of analysis could readily be extended to columns having unequal principal moments of inertia and of columns having pins inclined to axis of column as well as to each other. S. Levy, USA

1091. **Lin, Tung-Hua, Inelastic column buckling,** *J. aero. Sci.* **17,** 3, 159–172, Mar. 1950.

A complete analysis of column buckling in the plastic range with a known initial curvature is given together with a typical illustrative example for a steel column. A method is proposed for calculating the deflection curve of an inelastic column up to failing load, so as to obtain the maximum load and to show mechanism of column failure. Effects of reversal of strain in center of the column having a large deflection and load and of variation of tangent modulus of elasticity across the section are also discussed. Method of analysis presented is also applicable in solution of statically indeterminate structures having inelastic column members. S. K. Ghaswala, India

1092. **Habicht, Franz R., Calculation of buckling of foundation piles** (in German), *Planen u. Bauen* **4,** 5, 165–167, May 1950.

1093. **Dolberg, M. D., On the buckling modes of rods** (in Russian), *Dokladi Akad. Nauk SSSR*, **71,** 5, 839–842, Apr. 1950.

Shapes of the elastic curves are investigated which correspond to the first and higher critical loads of centrally compressed, elastic rods rigidly supported at intermediate points. The differential equation is written in its most general form for variable rigidity and variable axial force. A number of general properties are deduced, in seven "theorems," for the elastic curves which satisfy the equation and the corresponding conditions of support at the ends and intermediate points, and for the corresponding critical loads. G. Winter, USA

1094. **Massonnet, Ch., Considerations concerning rational specifications for the buckling of steel bars** (in French), *Ossature métall.* **15,** no. 7–8, 358–378, July 1950.

A comprehensive paper which includes a review of column theory (Euler, Engesser, von Kármán, Shanley); development of allowable loading formulas for columns with eccentricity, lateral loading, and combinations of both; a discussion of principles underlying the establishment of strength regulations for columns; a review of various proposals for expressing the initial eccentricity of a bar in terms of its length or other dimensions; some examples illustrating application of the French and Belgian regulations; an extension of the proposed method to include buckling by combined bending and twisting.

The underlying principle is use of the expression $P_E/(P_E - P)$ as a magnifying factor for the initial eccentricity. This permits the total bending moment to be calculated. The direct (average) stress is then added to the total bending stress to obtain the maximum stress, which must not exceed some prescribed value (yield point). To account for lateral loading (initial bending), the initial bending moment is increased by a modified value of the magnifying factor.

Resulting formulas appear to be reasonably accurate for materials with a sharply defined yield point. For materials which have a gradual transition from the elastic range the use of the "elastic" magnifying factor may be questioned and the choice of a suitable maximum allowable stress becomes difficult. F. R. Shanley, USA

1095. **Jakkula, A. A., and Stephenson, Henson K., Steel columns—A survey and appraisal of past works,** *Texas Engng. Exp. Sta.*, fifth series, 3, 6, bull. no. 91, 118 pp., June 1, 1947.

1096. **Carrier, G. F., On dynamic structural stability,** *Proc. Symp. appl. Math.* **1,** 175–180, 1949.

Author considers structures such as rods, plates, and shells which are concentrically loaded in their axis or plane by dynamic loads. He derives a general differential equation for this problem and applies it to find the differential equation for case of thin circular disk which is subjected to uniform radial pressure which varies with time. Reviewer remarks that differential equation(s) for given case can be written down directly by introducing time dependent load(s) and inertia term(s) in known differential equation(s) for elastic stability. P. P. Bijlaard, USA

1097. **Pearson, C. E., Bifurcation criterion and plastic buckling of plates and columns,** *J. aero. Sci.* **17,** 7, 417–425, 455, July 1950.

Author states that when a bifurcation of equilibrium paths occurs in a nonconservative system, the preferred path might not be determined by stability considerations. He proposes the reasonable postulate that behavior of a perfect column in plastic range (where such bifurcation occurs) is the limiting behavior of a column with a vanishing initial imperfection. Analysis of the Shanley column model on this basis corroborates Shanley's original buckling concepts: buckling (i.e., beginning of bending) of a perfect column occurs at the tangent modulus load, and the load must increase as the buckle grows. Numerical and graphical procedures for analyzing the post-buckling behavior of real columns are discussed.

Author goes on to consider Shanley-type plastic buckling of simply supported plates in compression on basis of a linear flow theory of plasticity. Results constitute an extension of the work of Handelman and Prager [*NACA T.N.* 1530, 1948; AMR **1,** Rev. 1329] who used the same theory of plasticity but considered only buckling with unchanging applied stress. Author's results are not in good agreement with experiment.

Reviewer notes that author's analysis yields the startling result that no matter how low the yield stress may be, plasticity can never reduce the buckling stress of a long simply supported plate to less than 75% of the elastic value. Consequently, in reviewer's opinion, plate buckling stresses calculated on basis of a linear flow theory of plasticity must be viewed with suspicion. Reviewer remarks, finally, that the results for Shanley-type plate buckling found by Bijlaard [*J. aero. Sci.* **16,** 529–541, Sept. 1949; AMR **3,** Rev. 1925] and Stowell [*NACA T.N.* 1556, 1948; AMR **1,** Rev.

805] on the basis of a simple deformation theory of plasticity agree quite well with experiment. B. Budiansky, USA

1098. Bijlaard, P. P., On the plastic buckling of plates according to flow theory, *J. aero. Sci.* **17,** 12, 810–811, Dec. 1950.

Criticism of a paper by Pearson [see preceding review]. Author points out that Pearson, with doubtful success, attempts to reduce the disagreement between Handelman and Prager's flow theory and test results by assuming that entire plate remains fully plastic. About 85% of this improvement is obtained by the arbitrary assumption of incompressibility, i.e., $\nu = 0.5$. Author indicates that this problem was fully treated in general in his earlier paper [*J. aero. Sci.* **16,** 529–541, 1949; AMR **3,** Rev. 1925], and that Pearson's results are a special case of this. He concludes that test results confirm deformation theory rather than flow theory. No mention is made of the Batdorf slip theory which attempts to overcome the discrepancies of the flow and deformation theories.

Harold Lurie, USA

1099. Thielemann, Wilhelm, Contribution to the problem of buckling of orthotropic plates, with special reference to plywood, *Nat. adv. Comm. Aero. tech. Memo.* 1263, 122 pp., Aug. 1950.

This is an English version of a German paper with minor corrections and remarks by the translator. The first part presents the theory of elasticity of orthotropic plates, with special reference to relationship between the elastic moduli and the angle between the load direction and principal stiffness directions. The second part deals with the stability equation of the general-orthotropic rectangular plate under uniform shear and axial loads, which is solved for a very long plate strip. Using an approximate method, buckling loads and buckling lengths in pure compression and pure shear are plotted against the stiffness values of the plate in the case of orthotropic plate strips, whose principal directions of stiffness are inclined at 45° with respect to the plate edges, and against the inclination of principal stiffness directions relative to plate edges for special stiffness values of plywood plates of various constructions. G. Herrmann, USA

1100. Cicala, P., On plastic buckling of a compressed strip, *J. aero. Sci.* **17,** no. 6, 378–379, June 1950.

This short note gives a calculation to show that in the plastic buckling of a compressed strip there is considerable decrease in the maximum attainable stress due to small initial deformations. A step-by-step integration procedure is employed in solving the incremental (or flow) equations of plasticity in predicting buckling loads of the plates. D. L. Holl, USA

1101. Sturm, R. G., Smith, L. W., and O'Brien, H. L., Allowable eccentricity of spherical heads convex to pressure, *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 533–538, July 1950.

Paper discusses buckling loads of spherical heads or shells, initially out-of-round, when subjected to external pressure. Charts are constructed for design characteristics of some distinct metals such as aluminum 38SO. There is no experimental confirmation of the given analysis. D. L. Holl, USA

1102. Cyr, N. C., and Hechtman, R. A., Lateral buckling of simply-supported I-beams, *Trend in Engng.* **2,** 3, 13–16, July 1950.

Investigation compares values of the nominal bending strength predicted by theoretical analyses with those values determined experimentally. Yielding in compression flange of 7-in. Jr. beams with Id/bt ratios between 600 and 1900, significantly reduced their resistance against lateral buckling. This yielding was due to the combined effects of bending in both the vertical and horizontal planes and to the torsional warping of the cross sections of the beam. Results of this program indicated that the yield point of the steel as well as the modulus of elasticity is necessary to define the buckling strength of the I-beams in this range of the slenderness ratio. From authors' summary

1103. Engel, Howard L., Lateral buckling of bars of monosymmetric open cross section, *Div. Engng. Mech. Stanford Univ.*, T. R. no. 8, 49 pp., June, 1950.

Reference is made to Technical Reports nos. 1 and 6 of this series in extending the theory of lateral buckling of bars of thin-walled open cross sections having a single axis of symmetry. Ideas of uniform and nonuniform torsion are reviewed, and dependence of the axis of rotation upon means of support of the bar and upon distribution of twist along bar is established. Several problems are solved by the Rayleigh-Ritz method using the stability equation derived, and are compared with those obtained otherwise. An approximate solution to problem of buckling of a cantilever bar of monosymmetric cross section loaded at shear center of its end cross section is obtained, and the results are compared with results of tests of similar angle sections.

Finally, an experiment on an angle section in combined bending and torsion with parallel axis is described, and a qualitative account of the behavior of the angle is given.

Th. Pöschl, Germany

Joints and Joining Methods

(See also Revs. 1067, 1180, 1189)

1104. Dolan, T. J., and McClow, J. H., The influence of bolt tension and eccentric tensile loads on the behavior of a bolted joint, *Proc. Soc. exp. Stress Anal.* **8,** 1, 29–43, 1950.

Paper deals with strains in a single bolt joint due to eccentric tensile load. Formulas for calculating strains in the bolted assembly are derived. Effect of varying initial tension, elastic properties of bolt, and eccentricity of applied load on strain distribution in the assembly are determined experimentally. Test specimen consisted of an assembly of three parts, each 2½ in. long, connected by a single ⅝-in. bolt through a ¾-in. hole. Clearance between bolt and assembly permitted use of electric strain gages on the bolt and assured its unrestrained bending. Good agreement between theoretical and experimental results was obtained. Both were limited by following conditions: (a) Loads in elastic range, (b) long bolts unrestrained by bearing contact between bolt and assembly, and (c) no shear load in bolt.

B. Bresler, USA

1105. Bergholm, A. O., Swartz, O. W., and Hoell, G. S., Stress distribution around spot welds, *Weld. Res. Suppl.* **15,** 5 217–223, May 1950.

Frequent failures at or around weld spots point to need for fundamental information on stresses at resistance welds. Paper deals with two and three spots tensile loaded in the line of the welds. Directions of principal stresses were determined by use of a photoelastic model and Stresscoat enamel. Electrical strain gages were then applied contiguous to welds. High compression strains measured on outer surfaces of plate where load entered the weld joint indicated bending and, consequently, high tension in the contacting surfaces of the lapped plates. Maximum tension occurred along the lines of applied force in regions between spot welds. Authors conclude that single laps of spot-welded joints are inherently weak due to unavoidable bending of the plates; that double laps will reduce stress concentration; that relief of stresses might be obtained by a drilled hole to redistribute

stresses near the weld (no experimental data on this point); and finally, that use of one row of resistance spots is undesirable in important structures carrying variable loads or reversal of stress.
Carl E. Hartbower, USA

1106. **Thielsch, Helmut, Weld embrittlement in chromium stainless steels,** *Weld. Res. Suppl.* **15, 3,** 126–132, Mar. 1950.

Furnace heat treatments, induction heating tests, and tests on weld deposits were made on four major types of alloys. Embrittlement seems to accompany severe grain growth, but size alone will not account for it. Hardness values alone give insufficient indication. Multibeads are superior to single bead welds. Post-annealing is far superior to any change in weld technique. Information on alloys studied, experimental procedures, results obtained, and conclusions drawn form the basis of paper.
Frederick K. Teichmann, USA

1107. **Nippes, Ernest F., Pfluger, Allan R, and Slaughter, Gerald M., Seam welding monel metal to steel,** *Weld. Res. Suppl.* 15, 3, 134–140, Mar. 1950.

Studies were made to avoid porosity and cracking in seam-welding of monel to steel. Optimum conditions were established whereby "braze" type welds proved superior to seam welds of steel to steel. Discussion of equipment, procedures, determination of weld on time and electrode force, weld off time, weld spacing, welding currents, together with several graphs of results and photographs, present the results of the investigation.
Frederick K. Teichmann, USA

1108. **Holt, Marshall, and Clark, J. W., A study of end connections for struts,** *Proc. Amer. Soc. civ. Engrs.* **74,** 1477–1499, Dec. 1949.

Results of static and fatigue compression tests of tubular and both single- and double-angle struts with usual types of end connections, as well as some types not in common use are given.
M. Gololobov, Czechoslovakia

1109. **Wilson, Wilbur M., Mun , William H., and Snyder, I. Sterling, Fatigue strength of various types of butt welds connecting steel plates,** *Ill. Univ. Engng. Exp. Sta. Bull.* 384, 57 pp., Mar. 1950.

Considerable data are presented on fatigue tests of partial-penetration E6010 butt-welds in $^{7}/_{8}$-in. thick A7 carbon steel with joints longitudinal and transverse to direction of stress. Objective was to determine strength of welds in thick plate under loads that produce a relatively low stress in joint. Girth seam of a cylindrical pressure vessel was cited as an example. Partial-penetration welds were investigated because their use under such conditions would represent a considerable reduction in cost. Specimens were 4 ft long, 1 ft wide at the ends, and 5 in. wide in the reduced section. S-N diagrams are presented together with computed values of fatigue strength corresponding to failure at 100,000 and 2,000,000 cycles of stress varied from zero to tension. Fatigue strength of longitudinal weld was nearly the same as for plate without a weld, while that of transverse weld was only 20 to 30% the strength of unwelded plate (in contrast to 75% for full-penetration transverse butt welds). The fact that the severe geometrical stress-raiser incident to the partial-penetration weld had little effect upon fatigue strength of longitudinal joint was attributed to the orientation of stress raiser with respect to direction of stress. Comparisons between $^{3}/_{16}$- and $^{1}/_{4}$-in. partial-penetration welds with and without weld reinforcement and between fatigue strengths for testing temperatures of 75 F and −20 F revealed only slight variations.

In a second group of tests on $^{1}/_{2}$-in. A7 plate containing single-V transverse butt welds, comparisons were made between E6010 and E6012 deposits with and without a high-temperature stress relief. Fatigue strength was very nearly the same whether welded with E6010 or E6012, with and without stress relief. A third group of tests were concerned with the effect of stress relief on $^{1}/_{2}$-in. A7 plate containing a longitudinal single-V butt weld, with and without weld reinforcement. Three methods of stress relieving were employed: high temperature (1150 F), low temperature (400 F), and mechanical (plastic flow). No material advantage in stress relieving was indicated.
Carl E. Hartbower, USA

1110. **Kuenzi, Edward W., Strength of aluminum lap joints at elevated temperatures,** *For. Prod. Lab. Rep.* 1808, 6 pp., Dec. 1949.

Tests at 6 temperatures from 75 to 600 F of lap-joint specimens with an overlap of $^{1}/_{2}$ in., made of 0.064-in. thick clad 24S-T3 aluminum alloy suitably etched and bonded with 9 different adhesives, 6 commercially available and 3 laboratory formulated. Strength was invariably found to drop with increasing temperature; highest strengths at higher temperatures were obtained with a hot-setting phenol resin-synthetic rubber adhesive (type M); the highest strengths (4500 psi) at room temperature were obtained with hot-setting modified-phenol-polyvinyl butyral adhesive. Failure was characterized by the percentage of glued area which failed in adhesion (i.e., where the adhesive separated from the metal surface) as distinct from area which failed in cohesion of the adhesive. Similarity in decrease of strength with rise of temperature in lap joints and sandwich constructions of the aircraft type [*For. Prod. Lab. Rep.* 1804] leads to conclusion that the former may be used to predict the latter.

It may be pertinent to add that the behavior of aircraft panel structures, in particular of panels consisting of a metal skin bonded to stringers, appears to depend on the "tearing" or "peeling" as well as on the shear strength of the adhesive, and that, as found by reviewer, peeling strength does not follow the same law of change as shear strength during an increase of temperature and may even increase. C. Mylonas, England

Structures

(See also Revs. 983, 984, 1016, 1032, 1081, 1091, 1175, 1390)

1111. **Egoroff, V. N., Practical method for hyperstatic structures calculation** (in Spanish), *Cien. Tecn.* **114,** nos. 571, 572, 573, 574, 1–22, 78–96, 109–140, 175–190, Jan., Feb., Mar., Apr., 1950.

Author shows how procedures involved in the method of fixed points (Strassner, Morsch, Suter, and others) and the method of primary moments (Efsen) may be systematized and simplified somewhat as applied to framed structures. Theory is outlined, procedure described, and a series of examples presented. A set of nomographs is included to assist in evaluating distribution coefficients, transmission coefficients, and factors for rotation and translation of joints. Properties of haunched beams for a wide range of proportions are given. Load factors for a number of common types of loading are tabulated. Solutions are given for several problems for which published solutions by other methods are available, enabling reader to compare author's procedure with other standard procedures. Glenn Murphy, USA

1112. **Kolař, V., Analysis of three-dimensional frame structures by energy method** (in Czech), *Technicky Obzor SIA* 58, 3, 39–43, Mar. 1950.

Analysis of frame structures as they occur in engineering practice in one- or multiple-story buildings with a structural skeleton

consisting of columns or walls running in two directions, in such a layout that torsional moments due to vertical and lateral loading reach considerable values. Reference is made to previous publications by Mayer on buckling problems, by Grammel and Pöschl on torsional stability, by Klouček on distribution of deformations, by Čiežk on torsion of frame girders, by Ostenfeld on method of deformations, and by Timoshenko. Two examples demonstrate the procedure. The second example (a water tank on elastic foundations) is characteristic of cases where deflection lines of the bottom slab depend largely on resulting distribution of internal forces, and are therefore unknown. It is assumed to follow a rational polynomial suggested by author. Comparison with an exact analysis by using Krylov's function and particular integral shows but a small difference (4%). Stability is investigated by Timoshenko's method. J. J. Polivka, USA

VIth Scientific Congress of the P.A.C.E.T.

The following material was contributed to the Congress in Gdańsk, Dec. 1–4, 1949 (Materialy nadeslane na VI. Zjazd Naukowy PZITB = Polski Zwiazek Inzynierów i Techników Budowlanych w Gdańsku 1–4 grudnia 1949) (in Polish), Edition of the Polish Ministry of Buildings No. 37, III. Wydawnictwo Ministerstwa Budownictwa Nr 37, cześć III:

1113. Hildebrand, E., The influence of sway on the knot's moment in multistory frames, 289–313.

Report establishes conditions in which it is permissible to neglect the influence of sway, which considerably simplifies the calculations of framed systems of structures. Author examines the magnitude of errors caused through such assumption with respect to number of spans, number of stories, stiffness of columns, and the way of connecting them with beams. He introduces the results as graphical diagrams and comes to some practical conclusions regarding the possibility of simplifying the calculations.
J. Mutermilch, Poland

1114. Olszak, W., Application of prestressing to columns with coiled reinforcement, 52–66.

Author considers problem of reinforced-concrete columns with prestressed coiled reinforcements and calculates them as anisotropical elements. The series of graphs, which introduce the results of research, show that the increase in carrying capacity of a concrete column with prestressed reinforcement gives greater effect with higher quality of steel and lower quality of concrete. When the standard dimensions of coiling are applied and by using steel with ultimate stress of about 120 kg/sq mm, prestressing of reinforcement can give considerable economy in the latter.
J. Mutermilch, Poland

1115. Olszewski, E., and Kowalewski, Z., Contribution to problems of three-dimensional systems of framed structures, 314–326.

A few simplifications, possible in some cases, in calculations of three-dimensional framed structures by the method of deformations. Authors point out that consideration of space in the calculations saves material, and show the possibility of working out tables which will simplify such calculations.
J. Mutermilch, Poland

1116. Wierzbicki, W., The buckling of three-pinned arches, 228–238.

Work has in view an approximate calculation of the critical force in a three-pinned arch of a small center rise and in which, as it is generally known, symmetrical form of buckling takes place. As in his other works discussing two-pinned and rigid arches, author applies the finite differences calculus.
J. Mutermilch, Poland

1117. Czulak, J., Statical calculations of circular arches by the method of least squares, 131–140.

Author introduces method of calculation of circular arches, which consists of substituting curves of parabolic type in Bresse's formulas of the curves which express the relationship between the arc differential *ds* and their position on the axis. The results, compared with those of an accurate calculation, give satisfactory conformity for practical purposes. W. Wierzbicki, Poland

1118. Naleszkiewicz, J., Study of periodic forces inducing vibrations in the foundations of machines, 194–214.

Study concerns case when a machine is fixed to the foundation by means of two rows of bolts, so that each row, during the time of vibration, creates a force which, when elastic elongation takes place in the bolt, fluctuates proportionally to it. Vibrations are considered which cause the base of the machinery to be lifted up from the surface of the foundation on one side and to be pressed in on the other. The fluctuation of forcing moment can come into resonance with any of the combined harmonic forces of its own vibrations, i.e., subresonance takes place. As an antidote against the creation of the subresonances, author suggests increase in total area of bolts and, also, in primary tightening of them.
W. Wierzbicki, Poland

1119. Błaszkowiak, S., Latticed bars eccentrically loaded, 260–268.

Author solves the title problem using Hardy Cross method. A multistory frame is investigated. Author compares his results with those of Muller-Breslau and obtains differences amounting to about 1%. W. Wierzbicki, Poland

1120. Błaszkowiak, S., and Kaczkowski, Zb., Contribution to the dynamics of trusses, 118–122.

Authors apply Hardy Cross method to calculation of the natural vibrations of trusses with rigid joints. They consider the fact that the weight of individual bars is distributed along their length, which generally has not been taken into account.
W. Wierzbicki, Poland

1121. Kolousek, V., Vibrations of statically indeterminate structures subjected to the action of moving, harmonically variable changeable loads, 503–504.

Author bases his work on the analysis of natural vibrations and applies this method to case of a bar which is subjected to the action of a live load moving with constant speed. When resonance takes place between the frequency of the live load and that of its own vibrations, the problem shows important characteristics. W. Wierzbicki, Poland

1122. Norris, C. B., and Kommers, W. J., Short-column compressive strength of sandwich constructions as affected by the size of the cells of honeycomb-core materials, *For. Prod. Lab. Rep.* 1817, 7 pp., Aug. 1950.

Describes tests on sandwich materials having aluminum facings of various thicknesses and solid core materials in which a cell of a honeycomb-core material was simulated by a round hole. Details of materials used, preparation of specimens, and test methods are also given. Results are presented in tables showing

values of critical stress for a variety of materials and core and face thicknesses. An empirical curve is obtained from which can be estimated the critical stress of the sandwich plate. The generalized data indicate that the critical or buckling stress of other facing materials can be estimated from these tests and from the approximate formula derived. S. K. Ghaswala, India

1123. **Boller, K. H., and Norris, C. B., Effect of shear strength on maximum loads of sandwich columns,** *U. S. Dept. Agric. For. Prod. Lab.*, no. 1815, 12 pp., June 1950.

In thin stressed-skin structural columns, such as sandwich columns having relatively weak cores, any deviation from the true shape (i.e., any initial buckling) may induce core-shear failure at a load smaller than that required to cause failure by (1) local instability of the facing (wrinkling), (2) over-all instability of the panels (critical buckling), or (3) direct compressive stresses greater than the strength of either facing or core.

Results of static tests on 50 sandwich columns, consisting of aluminum-faced cellular cellulose-acetate cores, confirm maximum edgewise loads computed according to formula developed for columns with known minute initial curvature and failing as a result of excessive transverse shear stresses. The formula calls for use of secant modulus of rigidity at failure of core material.

E. George Stern, USA

1124. **Fine, M., The warping and constraint of certain rectangular box sections,** *Aircr. Engng.* **22,** 256, 172–173, June 1950.

Author derives equations for computing the location at which a transverse force must be applied to a rectangular cantilever box beam to prevent rotation of end cross section. In a numerical example in which root of box is clamped from one side only, the transverse force must be shifted approximately one seventh of the interspar distance to neutralize rotation due to one-sided support at root. Samuel Levy, USA

1125. **Denke, Paul H., Analysis and design of stiffened shear webs,** *J. aero. Sci.* **17,** 4, 217–231, Apr. 1950.

Graphs are presented for the average stress distribution in buckled shear panels as well as for the local bending loads induced in longitudinal and transverse stiffeners, based on analysis in an earlier paper by author [title source, 11, p. 25, 1944]. Independent parameters are panel aspect ratio, longitudinal and transverse stiffening ratios, and shear stress to buckling stress ratio. Beyond the elastic limit an empirical relation between an effective web modulus of elasticity and the shear is applied. Comparison with NACA tests [*T.N.* 1364, 1947; AMR **1,** Rev. 634] shows fair agreement up to failure. W. T. Koiter, Netherlands

1126. **Solvey, J., The influence of wing geometry and structural efficiency on aircraft performance,** *Aero. Res. consult. Comm., Aero. Res. Lab. Melbourne, Austral., Rep.* ACA 45, 25 pp., Mar. 1949.

Four airplanes (one fighter, two civil transports, one bomber) are investigated to determine their wing-structural efficiency based upon a criterion established in a previous report [Solvey, "Structural efficiency of wing," same source, *Rep.* ACA 44, Jan. 1949]. By means of this criterion the influence of two parameters—variation of plan form and wing thickness—on structural weight and on performance is investigated. The expression for the original criterion is still too complicated to be used analytically, but a numerical investigation is feasible.

For the investigations made, it is found that plan form does not affect the weight of shear and bending material, nor is there more than a possible 1 to $2\frac{1}{2}\%$ variation in total weight of wing. Considering practical acceptable variations, change in weight for cases considered would not have been more than 10 pounds. In investigating the effect of wing thickness, defined by depth at root, and spanwise depth taper, it was found that total wing weight increases with increasing spanwise depth taper, whereas weight of members carrying shear and bending moment decreases slightly with increasing spanwise depth taper. The influence of the two parameters chosen on the performance indicated some or no changes in level speed, range, or climb.

Frederick K. Teichmann, USA

1127. **Symonds, P. S., and Prager, W., Elastic-plastic analysis of structures subjected to loads varying arbitrarily between prescribed limits,** *J. appl. Mech.* **17,** 3, 315–323, Sept. 1950.

A structure is not considered unsafe if it would suffer plastic flow of limited magnitude a finite number of times without collapsing. After limited plastic flow, structure is said to have "shaken down" to a state of residual stresses if for all further load applications no further plastic flow occurs in any part of structure. Authors show that if any state of residual stress exists, so that the resultant of the residual stresses and the elastic stresses induced by external loading (arbitrary combination of components within prescribed limits) remains within the yielding point, then structure eventually will shake down, even though the shake-down state is not unique. The question of the ability of the structure to shake down is then to find the existence of any such residual stress state. Criteria for shake down are derived. This theory gives a more realistic concept of "failure" to obtain the most economical design consistent with safety. Treatment is elegant. A geometrical representation of the instantaneous state of stress in an appropriate "stress space" is used. Present paper is based on a pin-jointed plane truss as a typical structure. The materials are either "ideally plastic" or with "linear strain-hardening."

Several forms of principle of virtual displacements are given from which the minimum principles of Haar-von Kármán and of Greenberg are derived. Authors point out that the Haar-von Kármán principle is valid, in general, only so long as those bars which have reached their yield forces do not cease yielding, but Greenberg's principle is valid for any loading sequence.

Three examples are given: A structure consisting of two concentric parallel bars; one with three parallel bars; and one with three bars meeting at a joint. The solutions are visualized geometrically in stress spaces. Y. C. Fung, USA

1128. **Tests of steel-girder spans on the Chicago & North Western Railway,** *Amer. Rly. Engng. Assn.* **52,** 488, 1–91, June-July 1950.

Report embraces a description and analysis of test data secured on 3-deck plate-girder spans varying in length from 70 ft $\frac{1}{4}$ in. to 78 ft on the Chicago and North Western Railway. Two of the bridges had an open timber floor while the other bridge had a ballasted precast-concrete floor. Stresses were measured by means of electromagnetic strain gages, with oscillograph recordings, in various parts of the bridges. Data secured during tests are analyzed for purpose of determining the static and total impact effects as well as the individual effects which contribute to total impacts. From abstract

1129. **Stüssi, F., A contribution to the calculation of anchored suspension bridges** (in German), *Inter. Assn. Bridge Struct. Engng., Fin. Rep.*, 3rd Congr., 483–491, Sept. 1948.

Author gives the diagram of a method of static calculation of anchored suspension bridges, based on the equation of the funicular polygon. A remarkable characteristic of this type of bridge is that the three regular forms of the equilibrium equation appear

simultaneously. Elimination of forces in hangers and deflections gives moments in stiffening girders. For a given value of the total horizontal stress in the cable, this equation is linear; it follows that the law of superposition can be applied and the method of calculation is same as that of statically indeterminate systems. Author also deals summarily with a few secondary influences. In conclusion, he refers to the "Ackeret-Egli" tests concerning influence of a longitudinal slot in the roadway on the aerodynamic stability of suspension bridges.

From author's summary

1130. Courbon, J., Calculating the flexible pylons of suspension bridges (in French), *Inter. Assn. Bridge Struct. Engng.* **9**, 69–82, Nov. 1949.

It is often possible to fix the feet of pylons of suspension bridges, since the stresses occurring in consequence of the displacements at the top, which are caused by deformation of the cables, can be taken by the big compressive forces without overstressing.

Several methods of making the calculations are given. The first three refer to pylons whose own weight is negligible and with a law for the run of the moments of inertia which enables the solution of the problems to be found in a consistent form. Two of the laws thus considered refer to a pylon with constant cross section and to a pylon which has the shape of a truncated pyramid. Two other methods make it possible to calculate a pylon whose weight cannot be neglected and whose moment of inertia varies in any desired manner. From author's summary

1131. Asplund, S. O., Deflection theory analysis of suspension bridges, *Inter. Assn. Bridge Struct. Engng.* **9**, 1–33, Nov. 1949.

A general, systematic influence line method for the analysis of variable stiffness suspension bridges is deduced.

From author's summary

1132. Beer, H., Stereometrical study of suspension cables (in German), *Inter. Assn. Bridge Struct. Engng. Fin. Rep.*, 3rd Congr., 493–506, Sept. 1948.

Paper deals with the general case of a cable suspended from two or more elastic points, under the effect of any kind of spatial loads, applying particularly to the study of erecting suspension bridges (cables and horizontal stresses). Author explains an iteration method which gives a rapid solution to all the cases encountered in actual practice. Position of the cable is calculated by assuming there are horizontal stresses; these are then determined graphically. A numerical example indicates the process for applying method. From author's summary

1133. Mitchell, G. R., Research on the strength of bridges. (c) Problems of impact and fatigue and their effect on permissible stresses in cast iron girder bridges, *Inter. Assn. Bridge Struct. Engng.* **9**, 61–68, Nov. 1949.

First part of paper describes experiments to measure dynamic stresses in girders of old cast-iron bridges using both tracked and wheeled vehicles for loading purposes. Results indicate that there is a range of values of the impact factor defined as ratio of dynamic to static stress for the same vehicle from about 0.7 to about 1.3 with a most frequently occurring value of about 1.0. Implications of results are discussed and an indication of further work to be carried out is given.

Second part deals with the problem of fatigue in bridges of this type and also, briefly, with the question of different factors of safety for dead and live load stresses. Stress in a bridge girder is considered as a steady dead load stress together with a live load stress which varies from zero to a maximum a large number of times during the life of the bridge. It is shown that, by looking at these stresses from point of view of fatigue, permissible total stress should vary according to value of dead load stress. Making use of suitable factors of safety, a formula for permissible live load stress in cast-iron bridge girders is given.

From author's summary

1134. Massonnet, Ch., The transverse distribution of loads in bridges with several adjacent arches (in French), *Inter. Assn. Bridge Struct. Engng.* **9**, 341–366, Nov. 1949.

A bridge with upper-lying track, supported on more than two parallel arches, is considered, and investigation is made as to how a load applied to the track is distributed on the different arches.

First, author tries to find the loads under which the track experiences a bending of the form $w = f(x) \cdot g(y)$. In this case transverse beams are deformed similarly and behave as beams on flexible supports. It can be shown that the distribution of the loads on the various arches depends only on one coefficient, termed the coefficient of transverse distribution. Author then shows how any desired loading of the track can be split up into a series of such particular loads. The maximum moments in the arch are obtained, sufficiently accurate for practical purposes, by loading one half of the bridge. It can therefore be stated that the loads are distributed as shown under the second special loading. The calculation of a bridge with several parallel arches therefore becomes similar to that of a bridge with several adjacent girders. Finally, the theory is applied numerically to an existing bridge (Pont de Neuilly). From author's summary

1135. Grinter, L. E., Stresses in gusset plates by use of an analogous grid, *Inter. Assn. Bridge Struct. Engng. Fin Rep.*, 3rd Congr., 265–276, Sept. 1948.

By properly proportioning an analogous grid of rigidly connected members meeting at right angles, physical deformations of a continuous plate due to any loading may be approximated with good accuracy. Analysis of stresses in such a rigid gridwork follows from author's previous publication of a method for analyzing wind stresses in a multistory building frame by moment and force distribution. The particular problem of a gusset plate is shown to be similar to any plate or wall problem where the forces exist entirely within plane of plate. An example of method, as applied to a typical gusset plate by Marvin Mass, is presented in an appendix. From author's summary

1136. Goldenblat, I. I., Some new problems of structural dynamics (in Russian), *Izvestiya Otd. tekh. N.*, no. 6, 819–833, June 1950.

A brief progress report of work on miscellaneous projects under way at the Central Scientific Research Institute for Structures. Items discussed include: (1) Stability of columns under combined static and dynamic axial loadings. Experimental apparatus is described for free and forced vibrations to check natural frequencies and régimes of stability. For static loads of 70% of Euler critical, free vibrations were highly nonlinear. With static loads up to 80% of critical, superimposed oscillatory loadings giving peak column loads up to 150% of critical did not result in instability. (2) Stability of structures under moving loads. Theoretical work of previous investigators is reviewed, pointing out certain errors. Analytical discussion of locomotives speeding over a bridge indicated the possibility of unstable lateral vibrations. Similar situation arises for fast flow of frictionless fluid through flexible tube. (3) Stability of suspension bridges. Considers the case of two axes of symmetry in the cross section of the road bed. By adjusting dimensions so that the natural frequency

of horizontal vibrations was equal to that of vertical vibrations, it was shown experimentally on a 5-m model that the energy of vertical vibrations was transferred into horizontal vibrations. Such vertical vibrations may be forced in a steady wind by shedding of a von Kármán vortex street. Other problems are mentioned without indicating that any work had been done.

Walter W. Soroka, USA

1137. **Cramer, H., Prismatic structures with transverse stiffeners,** *Concr. Const. Engng.* **45,** 3, 81–86, Mar. 1950.

From equilibrium conditions and conventional stress distribution assumptions, author develops an equation analogous to the three-moment equation, but involving shearing forces on three consecutive parallel edges of plane slabs. From shearing forces, normal forces, moments and stresses may be determined. Area moments, or a derivative, may then be used to evaluate flexural deflections. Effect of stiffeners is discussed and an example presented to illustrate details of calculation.

Glenn Murphy, USA

1138. **Thompson, J. Trueman, and Loewer, Alvin C., Jr., A new technique for bond measurement in reinforced concrete,** *Amer. Soc. Test. Mat.*, no. 166, 69–74, May, 1950.

Authors describe a segmented hollow bar containing electric-resistance strain gages for bond measurements in pull-out specimens. A few exploratory test results are given, which show that proposed technique is promising in obtaining the load-slip relation of reinforcement bars. Further work is required to enable a detailed study to be made under various experimental conditions.

G. G. Meyerhof, England

1139. **Whitney, Charles S., Application of plastic theory to the design of modern reinforced concrete structures,** *J. Boston Soc. civil Engrs.* **35,** 1, 29–74, Jan. 1948.

1140. **Kasarnowsky, S., On the deflection theory of arches** (in Swedish), *Inst. Brobyggnad. Medd.*, no. 3, 53–77, 1948.

First chapter deals with general theory of arches regarded as variable systems. Effect of horizontal deformations was not taken into account in calculation of bending moments since these deformations are of little importance, particularly in case of flat arches. Second chapter treats effect of deflection on horizontal thrust. Deflection of arch causes an increase in horizontal thrust. This increase is of no importance in calculation of normal stresses set up in the arch, but exerts a favorable influence on moment distribution, especially in flat arches. Third chapter considers moment distribution due to shortening of arch caused by compressive stresses, variation in temperature, shrinkage, or horizontal displacement of the abutments (parasitic moments). All investigations were made for parabolic arches of constant rigidity and for arches of nonuniform cross section represented by the Kungsbro (King's Bridge), Stockholm, Sweden.

From author's summary

1141. **Takabeya, Fukuhei, and Takabeya, Ryōhei, Characteristics of foundation reactions for the building frames, carrying loads vertical and uniformly distributed,** *Memo. Fac. Engng. Kyushu Univ. Fukuoka, Japan,* **12,** 2, 139–200, Sept. 1950.

Article is a continuation of authors' work on rectangular building frames. Vertical and horizontal reactions from vertical loads, and vertical reactions from horizontal loads are determined for multistoried frames wth columns fixed at their bases. No attention is directed to the moments at the bases. Solution is by a modification of slope deflection. Determination of the vertical reactions from vertical loads seems overelaborate when the probable actual variations in loads and soil conditions are considered.

Charles Clarahan, Jr., USA

1142. **Mazure, J. P., and van Exter, J. Ph., Common action of structure-parts in case of local loads in wooden roof structures** (in Dutch), *Ingenieur* **62,** 32, 95–101, Aug. 1950.

In the stress analysis of wooden roof constructions account must be taken of a concentrated load of 100 kg. For several members, this requirement determines the dimensions. Authors investigate the reduction in stresses and deflections caused by the cooperation between boarding and rafters. Two calculations are given, based on rather arbitrary assumptions about the distribution of internal forces; two approximate formulas, giving the reduction referred to above are derived, which are checked against an experiment. Reviewer must stress that the exact solution of the elasticity problem under consideration has been given by different writers using Fourier series, among them M. Hetényi in his book, "Beams on elastic foundation," pp. 192–197.

Ch. Massonnet, Belgium

1143. **Wenk, Edward, Jr., A frame for testing structural models,** *Proc. Soc. exper. Stress Anal.* **8,** 1, 67–78, 1950.

See AMR **3,** Rev. 1645.

1144. **Seide, Paul, and Barrett, Paul F., The stability of the compression cover of box beams stiffened by posts,** *Nat. adv. Comm. Aero. tech. Note* 2153, 29 pp., Aug. 1950.

Authors make limited investigation of new type wing structure in which top and bottom covers are connected by flexible posts. Energy method with Fourier series is used.

Charts are given for buckling loads of long box beams under end moments with 1, 2, or 3 rows of posts. Mode of buckling depends on design parameters and may be with longitudinal nodes through posts, transverse nodes through posts, or with displacement of posts. Longitudinal nodes are not obtainable if tension-cover thickness is equal or less than that of compression-cover.

Numerical example shows that buckling stress of 40,000 psi is obtainable with post weight of only $2^1/_2\%$ of compression-cover weight. Authors do not claim that such low weight ratios are obtainable in actual box beams but urge that possibilities of post-type construction are worth exploring.

P. C. Dunne, England

1145. **Magnel, G., Structures in precompressed steel** (in French), *Ossature métall.* **15,** 6, 300–313, June 1950.

Possible savings in weight and cost which may be obtained by prestressing are studied first for the case of a tension member. Before applying the load, bar is compressed by a cable to a working stress R^1 in compression. Supposing that a tensile load produces tensile working stresses R_l in the bar and R_c in the cable, the required areas of bar and cable are determined in terms of the applied force and R^1, R_l, R_c. From these, the total weight of the prestressed bar and cable can be compared with that of the simple bar, taking this to be loaded to produce a working stress R, and costs of the two structures can be compared. A factor of safety with respect to the elastic limit stress R_e is defined as the ratio $(R_e + R^1)/(R_l + R^1)$. For the simple bar the corresponding factor is R_e/R. By suitably choosing the ratio R_c/R the two safety factors can be kept the same, and author finds, for example, that economies in weight of 33% and in cost of 13% can be obtained by prestressing, assuming the ratio $R/R_c = 14/100$, and the price per foot of cable to be 3 times that of the bar. Higher economies are obtainable if safety factor is allowed to decrease slightly, and still further gains are had by prestressing in two or

more steps, while the load is applied. When the tension member is part of a girder, economy is affected by relative weights of member and total structure, as well as by ratio of dead load to total load.

Author applies these concepts to case of a lattice girder whose lower chord is prestressed by high strength wires. Measurements of stresses and deflections were made on a girder (span 14.32 m, height 0.936 m) with prestressing according to the author's theory. Satisfactory agreement between calculated and measured stresses and deflections was found. The economy in this case was not very high because of dimensions and load ratios chosen. Nevertheless, author's method will probably effect worth-while economies in certain types of structures and merits further attention. P. S. Symonds, USA

1146. Magnel, G., Prestressed steel (new considerations) (in French), *Ossature métall.* **15,** 9, 428–435, Sept. 1950.

Author cites examples indicating that the principle of prestressing steel members has been used in numerous structures in past half century. He extends his previous analysis of precompression of steel tension members (see preceding review) to cases where precompression is effected under dead load alone. Addition of live load then brings stresses in bar and cable to preassigned safe limits in tension. Solution and possible economies are discussed by means of diagrams. P. S. Symonds, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 1045, 1051, 1097, 1098, 1165, 1170, 1186, 1193, 1366)

1147. Stüssi, Fritz, The foundations of the mathematical theory of plasticity and the test (in German), *Z. angew. Math. Phys.* **1,** 4, 254–267, July 1950.

Author describes experiments to test "fundamental assumptions" of plasticity theory: (1) Zero plastic volume change, and (2) isotropy. He applies internal pressure plus end forces to thin hollow cylinder of aluminum alloy; measures circumferential strain ϵ_x and longitudinal ϵ_y by differential wire-roller-mirror attachments. Using stress σ_x alone he finds, in effect $\epsilon_x + 2\epsilon_y \neq \sigma_x/3k$ (k = bulk modulus); therefore rejects (1). Starting with σ_x in plastic range and, e.g., superposing σ_y, he finds strain increments are elastic, whereas they would be plastic for increment in σ_x; therefore rejects (2). Reviewer notes confusion between "anisotropy due to stress" [Prager, *J. appl. Mech.* **15,** 226, 1948; AMR **1,** Rev. 1466], which author has demonstrated but which even isotropic theory predicts, and anisotropy *of virgin state*, which is usually neglected but need not be [Hill, *Proc. roy. Soc. Lond. Ser. A*, **193,** p. 281, 1948; AMR **1,** Rev. 1121]. Also, author's test of (1) implicitly assumes $\epsilon_z = \epsilon_y$, i.e., isotropy, which he elsewhere rejects. William Fuller Brown, Jr., USA

1148. Hodge, P. G., Jr., Yield conditions in plane plastic stress, *J. Math. Phys.* **29,** 1, 38–48, Apr. 1950.

Assuming perfect plasticity and plane stress, author considers two cases of plastic flow: (1) Hole in infinite plate subjected to biaxial tension at infinity and radial internal pressure at edge, and (2) notched strip in tension. Besides the Tresca (maximum shear) and von Mises (shear energy) yield conditions, author introduces "generalized Tresca" yield condition containing parameter λ such that for $\lambda = 1$ the Tresca condition is obtained. By varying λ, effects on solution of small changes in yield condition may be estimated. For case (1), results indicate that distribution of radial stress is not very sensitive to such changes in yield condition. For notch, case (2), solutions are presented for generalized Tresca and von Mises criteria; it is indicated that for this case, slight changes in yield condition will make a pronounced difference in the solution. Incidentally, results obtained for notch indicate maximum load to cause flow at notch semi-angle of about 70° for idealized material satisfying von Mises criterion.

A. M. Wahl, USA

1149. Thorne, C. J., On plastic flow and vibrations, *J. appl. Mech.* **17,** 1, 84–90, Mar. 1950.

Solutions are presented for a mechanical model comprising a mass supported by a spring in parallel with a spring-dashpot combination, the mass being acted on by a force $F(t)$ where t = time. Formulas are worked out in detail for the following cases: (1) Constant force F_0; (2) alternating force $F = F_0 \cos \omega t$; and (3) force increased at constant rate $F = F_0 t$. Such a model has been utilized by Eyring and others in analyzing stress-strain diagrams of organic plastics and textiles. However, in discussion of paper [title source, **17,** p. 343, Sept. 1950] it is pointed out that model cannot represent plastic flow with large permanent strains in such a material as medium carbon steel (cited as an example in paper). A. M. Wahl, USA

1150. Süray, S., On families of curves attached to elastic or plastic bodies (in French), *Comm. Fac. Sci. Univ. Ankara* **1,** 33–40, 1948.

Author discusses those two-dimensional problems of elasticity and plasticity for which the lines of principal stress form isothermal nets. In the elastic case this question was first discussed by Neményi [*Z. angew. Math. Mech.* **13,** 364–366, 1933]; in the plastic case, by Carathéodory and Schmidt [*Z. angew. Math. Mech.* 3, 468–475, 1923]. Method used here differs from the methods used in these earlier papers in so far as it permits discussion of the elastic and plastic problems along analogous lines.

W. Prager, USA

1151. Vitovec, F., On the magnitude of Poisson's ratio in post-elastic deformation (in German), *Schweiz. Arch.* **16,** 3, 85–87, Mar. 1950.

Assuming that volume is constant, author discusses Poisson's ratio for deformations beyond the elastic limit. He shows that, when extensional strain is defined as change of length divided by original length, Poisson's ratio equals $^1/_2$ only for infinitesimal strains (elastic strain components neglected). For so-called natural or logarithmic strain, Poisson's ratio is $^1/_2$ even for finite plastic deformations (elastic components neglected).

Merit P. White, USA

1152. Roop, Wendell P., The part of octahedral theory in the study of the plasticity of metals, *J. Franklin Inst.* **249,** 3, 223–236, Mar. 1950.

Paper is a statement of belief in the simplest deformation theory for conditions of constant principal stress directions and for calculations of dissipated energy. Also, the octahedral shearing stress is suggested as a criterion of fracture. Reviewer suggests that the references quoted be read carefully by any newcomer to plasticity to understand the often strong limitations of the useful concepts advanced. D. C. Drucker, USA

1153. Hill, R., On the inhomogeneous deformation of a plastic lamina in a compression test, *Phil. Mag.* (7), **41,** 319, 733–744, Aug. 1950.

By using the Levy-Mises plastic stress-strain relations, a theory is developed for calculation of stresses and displacements in the problem of plastic compression of noncircular sheets. Experimental data are given in support of the theory. Two idealized

cases are treated and for each an analogy is made with a problem in elastic theory of plane strain. Authors suggest that the analogous elastic problems might be investigated photoelastically, although certain difficulties would be encountered. Use is made of Saint Venant's principle for analogous elastic problems to estimate values for plastic problems in question. Equations are developed and experimental procedures suggested whereby the coefficient of friction for cold-working processes may be determined. Gerald Pickett, USA

1154. **Cottrell, A. H., and Aytekin, V., The flow of zinc under constant stress,** *J. Inst. Metals* **77,** part 5, 389–422, 1950.

Experiments on single crystals and polycrystals show that creep can be resolved into transient and steady-state components. Transient component follows $(\text{time})^{1/3}$ law. Steady-state component varies exponentially with stress and with reciprocal of absolute temperature. Nature of steady-state component is examined, theory and additional experiments suggesting a balance between strain hardening and thermal softening.

J. C. Fisher, USA

1155. **Johnson, A. E., The creep of a nominally isotropic magnesium alloy at normal and elevated temperatures under complex stress systems,** *Metallurgia* **42,** 252, 249–262, Oct. 1950.

Creep properties of an initially isotropic cast magnesium alloy (98% Mg, 2% Al) were studied at 20 C and 50 C employing tubular test pieces (0.5-in. external diam, wall thickness 0.030 in. ±0.0005 in. over a 2-in. parallel portion) and pure tensile, pure torsion, and complex stress tests.

For low and moderate stresses, behavior was closely isotropical at both temperatures, anisotropy occurring at higher stresses. The Hencky criterion of plastic strain was obeyed over the entire stress ranges investigated. Equations are derived expressing the observed stress-strain relations over the temperature range concerned. Hans F. Winterkorn, USA

1156. **Garrod, R. I., Residual lattice strains in mild steel,** *Nature* **165,** 4189, 241–242, Feb. 1950.

This preliminary report covers x-ray diffraction experiments by author to clarify the existence of stresses left in individual grains of a polycrystalline aggregate after plastic deformation. Residual lattice strains were found to result from applied stresses well below the nominal yield. As an example, a certain family of grains exhibited not only negative residual strain after breakdown above the yield (in agreement with Greenough), but also exhibited positive values for lower applied stress.

A point of interest, however, was that no progressive disorientation in given families of grains at differing levels of applied stress, as expected from the Heyn theory, could be found. All patterns indicated uniform breakdown of the grains for the same value of previously applied stress. Charles W. Gadd, USA

1157. **Charron, Fernand, Friction and viscosity of plastic bodies** (in French), *C. R. Acad. Sci. Paris* **231,** 14, 646–647, Oct. 1950.

Paper deals with motion of easily deformed bodies, such as grease lubricants. Author treats boundary-layer contact problem. Experiments employ a vaseline-aluminum particle aggregate between first, two highly polished parallel glass plates, then a substituted unpolished lower plate; lower being fixed, upper moving parallel to it. Stereoscopic binocular microscope was used for observations.

Author notes, in case of unpolished lower plate, that upper plate, when moving slowly, does not disturb vaseline. Increasing the velocity of upper plate (and beginning with a certain limit) produces a drawing away of layers adjacent to it; velocity of layer depends on its depth.

Explanations are found in terms of the equilibrium equation written for the faces of the boundary layer. To write this, the previously developed relation for the force per unit of surface area, $X = f + \eta\, \partial u/\partial z$ is used. Layers are assumed to be displaced parallel to xy plane, motion is in x direction with velocity u; f is coefficient analogous to that of friction, and η the viscosity coefficient.

If one wishes to assume, as limiting conditions in the equations of motion, that the velocity of the lubricant in contact with the boundary is the same as that of the boundary, it is conclusively stated that the surfaces of contact must be unpolished. Author's ultimate objective is complete study of f and η for certain bodies under pressures from 1 to 1000 atmospheres.

J. Miklowitz, USA

1158. **Reiner, Markus, Twelve lectures on theoretical rheology,** Amsterdam, Holland, North Holland Publishing Co., 1949, 162 pp. $4.

Subject is presented in clear, concise manner without compromising the mathematical elegance of the tensor approach to problem. Treatment will seem elementary to those experienced in mathematics, but those primarily interested in fields other than mechanics may find the book somewhat advanced. From kinematics and dynamics, the discussion proceeds through various idealized bodies whose properties represent simplifying restrictions on the rheological equation involving stress and strain tensors. Latter part of book deals with nonlinear bodies, strength, rupture, plastic flow, dissipation of strain energy, and the more complex systems of sols and suspensions. Each lecture is followed by a series of notes where various points are amplified.

Principal feature of book is the correlation of behavior of many diverse materials in a general theory. Concept of strain energy and its dissipation lead to valuable applications to nonlinear systems which are of considerable engineering importance. Discussions are thought-provoking and well documented. Book should be valuable to all who are interested in applied rheology and indispensable to those who specialize in theoretical rheology.

John W. Cook, USA

1159. **Reiner, Markus, Deformation and flow—an elementary introduction to theoretical rheology,** New York, Interscience Publishers, Inc., 1949, xix + 346 pp. $6.50.

Book is an expansion of author's "Twelve lectures" (see preceding review) and includes a rich store of historical data dispersed throughout the text in a pleasant and readable fashion. Opening chapters deal with basic rheological concepts which are treated in detail. Chapter on viscosity contains fundamental data on instruments and methods of measurement. Brief discussion of plasticity is included for purpose of completeness. Structural viscosity of dispersed systems is considered extensively, bridging the space between Einstein's work on small rigid spheres to modern problems in macromolecular and polymer systems. Analysis of such systems is described, as well as the "power laws" used to describe experimental data. Wall effects are considered and methods are given for eliminating these factors. Chapter on volume elasticity and viscosity differentiates between these properties and the corresponding shear properties, leading to some discussion of strain work and energy dissipation under conditions of isotropic pressure. Several chapters are devoted to simple tension, bending, and torsion in which the customary treatment is given. Work-hardening is discussed briefly. A large amount of data is given for creep phenomena in concrete and mortar. Book

concludes with discussions of complex systems, theories of strength, and the second-order phenomenon of dilatancy.

Throughout, the approach is phenomenological, the atomic-molecular view being set aside for reasons given by author. In doing so, he has created a broad exposition of rheology which would have been impossible otherwise. However, his concepts of structural viscosity and complex systems lead to development in molecular terms in a logical manner. Book is extremely well documented and should be valuable as a reference as well as a source of ideas for further research. John W. Cook, USA

1160. Tyabin, N. V., Fundamental equation of the rheology of a Maxwellian fluid (in Russian), *Zh. eksp. teor. Fiz.* **19**, 6, 559–560, June 1949.

Author considers the motion of a Maxwellian fluid, for which

$$\dot{e} = \dot{P}_0/2G + \bar{P}_0/2\eta,$$

where $\dot{e}$ is the deviator of the deformation velocity, P_0 is the deviator of the stress, $\dot{P}_0$ its time derivative, G the rigidity modulus, and η the viscosity. For the case of an incompressible fluid he derives the equations of motion in the form

$$\eta\Delta v = \rho\frac{dv}{dt} + \nabla(p + U) + \tau\rho\frac{d^2v}{dt^2} + \tau\frac{d}{dt}\nabla(p + U)$$

where v is the velocity vector, p hydrostatic pressure, U potential of body force per unit mass, ρ density, and τ relaxation time, defined by $\tau = \eta/G$. Reference is made to J. Frenkel ["Kinetic theory of liquids," Oxford, 1946] for an earlier derivation of this generalization of the Stokes-Navier equations in a different form. In the case of steady flow described in a Cartesian coordinate system by the conditions $v_y = v_z = 0$, $v_x = v_x(y)$, equations reduce to those obtained in the case of an ordinary viscous fluid. Author also sets up criteria for similarity of two states of flow in case where the body force is that of gravity.

Courtesy of Mathematical Reviews N. Rosen, USA

1161. Kê, T'ing-Sui, Internal friction of metals at very high temperatures, *J. appl. Phys.* **21**, 3, 414–419, May 1950.

Studies of internal friction of metals up to 600 C have shown a friction peak (versus temperature) associated with the viscous behavior of grain boundaries. Besides this, an increase in friction with temperature is noted which is found to occur in single crystals also. This additional friction appears to be a common feature of all metals and has been observed by others.

Experiments with a torsion pendulum at a low stress level and a frequency of one cycle are reported on aluminum of various degrees of purity and cold work. The additional high temperature internal friction in both polycrystalline and single crystal specimens is found to: (1) Increase with amount of cold-work specimen was subjected to before recrystallization; (2) decrease with annealing at successively higher temperatures until a stable value is reached; and (3) increase with the precipitated impurity content. Observations support the view that friction is caused by presence of dislocations in the grain interiors. Dislocations are introduced by cold-working and some of them remain after complete recrystallization. J. M. Robertson, USA

1162. Read, W. T., Jr., Stress analysis for compressible viscoelastic materials, *J. appl. Phys.* **21**, 7, 671–674, July 1950.

Alfrey's theory of nonhomogeneous stress in viscoelastic media [*Q. appl. Math.* **2**, p. 113, 1944] and Mindlin's theory of photoviscoelasticity [*J. appl. Phys.* **20**, p. 206, 1949; AMR **2**, Rev. 843] are extended to include compressible media, inertia terms and any type of boundary condition. Correcting an error in Mindlin's paper, author shows how standard photoelastic techniques can be used to determine orientations of the axes of principal stress and the principal stress differences, even when the secondary principal axes of stress or strain do not coincide with the polarizing axes and the relative orientations of the three sets of axes vary with time.

R. D. Mindlin, USA

1163. Gray, V. R., The flow optical properties of visco-elastic media, *Proc. int. rheolog. Congr. Holland*, II, 23–31, 1948.

From optical studies of solutions of aluminum soaps in benzene, the following characteristics of flow birefringence in viscoelastic dispersed systems are drawn: (1) Initial birefringence; (2) an extinction angle falling with rate of shear; and (3) an increase of birefringence with rate of shear. All these go together with non-Newtonian viscosity. The liquid in steady laminar flow is considered as a photoelastic solid, while at the same time the continuously changing relative positions of the dispersed, either spherical or anisometric, particles is taken into account. Two limiting cases are often approached, viz. (a) dilute suspension of anisometric strongly birefringent particles in a liquid of nearly spherical, weakly birefringent molecules; (b) a network structure of strongly birefringent particles which retains a mean structure even at high rates of shear, swollen by a weakly birefringent liquid of spherical molecules. The birefringence of the systems is made up of two parts: one due to orientation of dispersed particles or units of structure, the other a photoelastic effect due to deformation both of dispersed phase and dispersion medium. Birefringence in a fibrillar liquid subjected to simple shear is considered in detail. M. Reiner, Israel

1164. Roscoe, R., Mechanical models for the representation of viscoelastic properties, *Brit. J. appl. Phys.* **1**, 7, 171–173, July 1950.

Author shows that any spring-dashpot model is equivalent to a two-terminal electrical network containing resistors and capacitors. He quotes a theorem of Cauer [*Arch. f. Elect.* **17**, p. 355, 1926] that any complex network is equivalent to either of two "canonic" forms. He gives a simple example, but does not indicate a general procedure for effecting this simplification.

Phillip G. Hodge, Jr., USA

Failure, Mechanics of Solid State

(See also Revs. 1154, 1176)

1165. Torre, C., Limit conditions for brittle failure and plastic behavior of ductile metals (in German), *Öst. Ing.-Arch.* **4**, 2, 174–189, 1950.

Author accepts a paraboloid of rotation expressed by: $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1) + 2(c-1)(\sigma_1 + \sigma_2 + \sigma_3) = 2c\sigma_z^2$ as the boundary surface for the initiation of plastic deformation. The ratio c of the stress at uniaxial compression (σ_d) and stress at uniaxial tension (σ_z) at equal strains is supposed to be >1 and $<2/3$, except at the initial yield stress where $c = 1$. Hence c is a function of strain and also different for different metals. Author suggests this condition, which is applied to several cases of loading such as torsion, axial tension, and compression with balanced transverse compression, plane tension, and triaxial tension in connection with the brittle fracture of welds, to be in reasonable agreement with experiments. J. H. Palm, Holland

1166. Stodden, K. J., and Klier, E. P., Brittle failure and size effect in a mild steel, *Weld. Res. Suppl.* **15**, 6, 303–310, June 1950.

An effort is made to show that under certain experimental con-

ditions the phenomenon of size effect can be sensibly eliminated as it pertains to the appearance of brittle failure. The specimen sizes studied ranged from the 10 mm × 10-mm Charpy bar to a 1¼ in. × 1¼-in. bar tested in slow bending. For these specimen sizes the transition temperature for a mild steel was invariant.

From authors' summary by C. O. Dohrenwend, USA

1167. **Agnor, T. J., and Shank, M. E., Fracture modes in high purity metals,** *J. appl. Phys.* **21,** 9, 939–940, Sept. 1950.

Brittle fracture impact with considerably lowered energy absorption found for ferrous materials at critical temperatures is now observed in high purity zinc. Macroscopic appearance of zinc and ferritic steel fractures is similar. Zinc fracture showed microscopically that large areas of ductile fracture occurred alongside areas of brittle fracture and deep voids. Authors conclude that transition temperature in metals is unrelated to aging and inhomogeneous yielding. These experimental observations may stimulate further research on fracture mode in engineering metals. K. H. Swainger, England

1168. **Enzian, G. H., and Salvaggio, G. J., The effect of nitrogen on brittle behavior of mild steels,** *Weld. Res. Suppl.* **15,** 11, 537–544, Nov. 1950.

Using various arbitrary criteria, based on Charpy and similar tests, the effects of nitrogen content between 0.004% and 0.011% on the tendency of mild steel to become brittle at low temperatures were determined. The largest nitrogen content used caused anywhere between zero and 50 F decrease in "transition temperature" below that for the smallest nitrogen content, depending on the criterion used. J. D. Lubahn, USA

1169. **Peterson, R. E., Nature of fatigue of metals,** *Mech. Engng.* 72, 5, 371–375, May 1950.

An interesting account is given of early English speculations and studies of the mechanism of fatigue, and subsequent work up to the present is traced. Comparisons drawn in paper between modes of failure in widely different types of materials, and the rather complete bibliography included, should be helpful to those interested in further research in this field. C. W. Gadd, USA

Material Test Techniques

(See also Revs. 1138, 1155, 1190, 1196, 1197, 1198)

1170. **Palm, J. H., The theoretical background and interpretation of the Schnadt-impact-test,** *Metalen.* **4,** nos. 9, 10, 11, 20 pp., May, June, July 1950.

In the Schnadt impact test a notched beam, similar to the Charpy specimen, is modified by placing a hardened steel pin in a hole parallel to the notch and on opposite side of beam. This pin is intended to produce triaxial stresses in the fracture region. Part I develops relations between principal stresses and normal and shearing stresses on the 3 planes bisecting the angles between the principal planes, and also on the 4 octahedral planes which make equal angles with the 3 principal axes. Similar relations are developed for strains. Two criteria for indicating start of plastic deformation are discussed, based on maximum shearing stress and on shearing stresses on octahedral planes, latter being preferred. Part II outlines concepts and system of Schnadt's tests. Triaxial stresses are classified according to ratio of octahedral shearing stresses and maximum normal stress and a similar relation for strains. This stress ratio is assumed to vary from 0.866 to 0.433 as notch diameter varies from infinity (no notch) to zero (sharp notch). Effect of temperature is discussed and some applications to practice are indicated. Part III covers theoretical background of the test. Value of hardened pin and quantitative accuracy are questioned. A postscript introduces additional test classifications based on Saint Venant's defined as $100d/(d + 2)$ where d is notch diameter. A bibliography is included. Several typographical errors were noted. W. B. Stiles, USA

1171. **Mintrop, H., Measurement of the large impact forces** (in German), *Schweiz. Arch.* **16,** 4, 119–124, Apr. 1950.

Author's method of measurement of impact forces is based on Hertz' classical equations, concerning the contact between elastic, solid bodies. In order to verify their validity and utility for this purpose, extensive static and dynamic tests were made, where the contact areas between spheres and plane solid surfaces were measured and the corresponding forces observed and computed. Balls were dropped on plane surfaces, and new methods, one of them involving the use of high speed films, were used to measure the time of impact and the diameter of the circular contact surface. On basis of these tests the Zeiss-Ikon Company has developed a very simple device, where a spherical segment on a cylindrical stem is pressed against a flat surface. An accuracy of ±2% is claimed for this device. Equipment used for these tests was destroyed and the records were lost in World War II. Bibliography cites 22 references. O. R. Wikander, USA

1172. **Banks, Luen B., Machine for testing rails in bending fatigue,** *Engineering* **169,** 4400, 585–587, May 1950.

A 15-ft length of railway rail with additional masses attached is vibrated as a free-free beam to constitute a full-scale fatigue test. Rubber supports are used at the nodes. It is driven by a rotating out-of-balance mass oscillator. Amplitude control is electrical, by varying the frequency on the side of the fundamental resonance peak.

It is found that high tensile steel rails in the black condition give the same fatigue strength as mild steel due to surface conditions.

Reviewer believes the theory to be in error, since attached masses are assumed to be equivalent to external forces. They should occur not with the forcing terms but with the vibrating system terms, and they will modify the resonance frequency, which will accordingly differ from that given, which is for a simple uniform beam without masses. E. H. Lee, USA

1173. **Found, George H., and Pittsley R., Residual stresses in magnesium: technique and results,** *Proc. Soc. exp. Stress. Anal.* **8,** 1, 1–13, 1950.

Knowledge of residual stress conditions in magnesium castings is needed for efficient design. Simple reliable method of determining residual stresses was developed. After SR-4 gages were cemented in place, small blocks were cut from critical sections using jeweler's saw. Cutting stresses were eliminated and thickness of blocks reduced by etching in chromic acid. Strain gage and original surface were protected with paraffin.

Magnesium alloys, as cast, develop residual stresses which appear to be reduced inversely as the temperature of heat treatment. Proper heat treatment leaves negligible stresses. Localized residual tensile stresses induced by solution heat treatment have low energy levels. Shot peening and burnishing produce relatively high residual compressive stresses in subsurface as well as surface. Reviewer believes techniques to be an improvement and the results to add to understanding of residual stresses in magnesium. C. R. Freberg, USA

1174. **Eitel, W. W., and Potter, J. H., A high-temperature extensometer,** *Instruments* **23,** 5, 448–449, May 1950.

In a high-pressure high-temperature flanged pipe joint, it is im-

portant to know bolt stresses at assembly and during operation. Paper describes mechanical extensometer designed to measure over-all bolt length. It consists of a "scissors" which spans the flange with sufficient clearance to allow for insulation. Readings are taken on a dial gage. The frame itself provides 3:1 magnification. Temperature effects are minimized by using silica rods in contact with the bolt and invar tubing for frame. Calibration against a proven standard showed maximum error of 0.68% in the range 200 to 900 F. B. F. Langer, USA

1175. Haviland, J. K., and McCaffrey, G. F. W., A hydraulic static testing apparatus, *Aircr. Engng.* **22,** 254, 100–103, Apr. 1950.

Description of apparatus in the Structures Laboratory, Div. Mech. Engng., Ottawa, for testing aircraft wings up to 120-ft span and 18-ft 9-in. chord. Test structure consists of twelve gallows frames, set up in the appropriate position on a reinforced-concrete strong floor.

Specimen is loaded by hydraulic tension jacks and held down by two pairs of anchorage beams. Description covers jacks and hydraulic consoles, tension patches normally used for applying loads from lever system to test specimen, deflection indicating system, calibration of jacks, etc. Brief discussion on preparation of wings of various types for testing is included.

F. J. Plantema, Holland

1176. Fusfeld, Herbert I., and Feder, Josephine Carr, Study of crack propagation using high speed motion pictures, *J. appl. Phys.* **21,** 3, 261–262, Mar. 1950.

Propagation of the tensile failure crack of certain materials was photographically observed. Relative motion of testing machine heads was 960 in./min with observations at the rate of 10,000 frames/sec. Cross-sectional dimensions of specimen tested are $^1/_2$ in. $\times$ $^1/_{32}$ in. Graphical correlation is given for crack length versus time, and velocity of crack propagation versus square root of crack length. No attempt is made to correlate initial or average rate of strain with above-mentioned quantities.

George H. Lee, USA

1177. Nielsen, L. E., Some instruments for measuring the dynamic mechanical properties of plastic materials, *Bull. Amer. Soc. Test. Mat.* no. 165, 48–52, Apr. 1950.

Author describes apparatus and experimental technique for determining the dynamic elastic moduli and damping or energy dissipation factors for vibrating specimens of plastic material. Two types of test are described. One uses the cylindrical specimen as the elastic element of a torsion pendulum. From observations of the free oscillations of the system, the shear modulus and logarithmic decrement are calculated. Frequencies in the range $^1/_{50}$ to 2 cycles have been used. The other test utilizes a specimen in form of a reed or strip forced to vibrate as a cantilever by the motion of the base, which is driven by a loudspeaker coil at frequencies of about 10 to several hundred cycles per sec. Observations of the amplitude through the resonance range enable Young's modulus and the damping coefficient to be determined. Author discusses applications of these test methods to studies of properties of plastic materials, such as polystyrene and polyvinyl chloride. P. S. Symonds, USA

1178. Weir, C. D., Caustic cracking: Stress-corrosion tests in sodium hydroxide solutions at elevated temperatures, *Inst. mech. Engng. appl. Mech. Proc.* **163,** 18–26, 1950.

Paper describes a series of tests carried out using an apparatus specially designed for stress-corrosion investigations at high temperatures. Specimens, which were of hollow form, were inserted into the base of an autoclave and loaded by means of a lever system through a push rod inserted in specimen. Arrangement permitted production of accurate notch-forms on external surface in contact with sodium hydroxide solutions.

It was found possible to produce failure of notched specimens rapidly and consistently, but homogeneously stressed specimens were immune. Fractures were intercrystalline and typical of those occurring in practice. Dilute solutions, though not entirely impotent, were found to be very much less effective than those of high concentration. Presence (or absence) of silica in solutions exerted no appreciable influence. Neither addition of tannin nor use of fine-grain steels was completely effective in preventing intercrystalline failure, though in case of the latter, increased resistance was noted in some cases. Cathodic polarization was found to be protective while anodic polarization did not prevent, and possibly hastened, failure.

From author's summary by T. J. Dolan, USA

1179. Ultrasonic testing of welds, *Engineering* **169,** 3289, p. 264, March 1950.

1180. Mironoff, N., Application of a new method of micro-mechanical testing of metals to weld tests (in French), *Ossature métall.* **15,** 6, 293–297, June 1950.

Mechanical properties of the metal in and near a weld are explored by indentation tests and by repeated bending tests of "microspecimens" which may be removed from the weld metal at intervals as small as 5 mm. Technique for test was described in the same periodical for 1945, no. 3/4. Test gives number of cycles N of bending through a given large angle required to break the specimen. Author describes in detail an exploration of welds in chrome molybdenum steel subjected to several heat treatments. He finds that the Brinell hardness number in the weld is proportional to $N^{-1/3}$ except in local regions where microstructure is such as to combine relatively high ductility with relatively high hardness or vice versa, low ductility with low hardness.

W. Ramberg, USA

1181. Parker, R. C., and Hatch, D., The static coefficient of friction and the area of contact, *Proc. phys. Soc. Sect. B.*, **63,** part 3, no. 373B, 185–197, Mar. 1950.

There exist various theories about the nature of friction, the older that friction is due to interlinking of the surface irregularities, the newer that cohesive forces play an important role.

With elastic materials, the real area of contact is proportional to the normal load. In the experiments ductile materials such as lead and indium were used to enable the area of contact to vary without changing the normal load. The metallic friction member was in the form of a hemisphere, the other consisted in a glass optical flat, through which the area of contact could be observed with a microscope. The apparent area of contact is distinguished from the real area of contact, latter being immeasurable and formed through the real points of contact.

From the experiments, following conclusions could be derived: (1) Tangential frictional force is proportional to apparent area of contact. (2) It is clear that, if cohesive forces are contributing to the friction force, this force must be proportional to real area of contact. The only explanation lies in the fact that there exists a constant ratio between the apparent and real area of contact. This assumption is made plausible with some experiments. 3 As the contact area grows with an increasing tangential load, the appearance of the contact area was studied with an electron microscope. This revealed that the asperities of the surface only began to yield if some tangential load was applied. With further increase in tangential load, a proportional increase took place both

in slip and in real area of contact, detectable through the appearance of little scratches on the tips of the asperities. After the whole apparent area of contact was covered with scratches, the tangential load reached its maximum value and the two parts slipped entirely ("macroslip"). No indication is observed for the theory of formation of welded junction.

It must be observed that the experiments were carried out with lead and indium; therefore it is not quite certain that the results and conclusions are entirely correct for harder elastic materials. Reviewer has performed a number of tests to determine the coefficient of friction between the surfaces of mild steel plates with various surface treatments. In these experiments the changes in the normal pressure could be studied during slip and macroslip [*Ingenieur*, **59**, p. 0.1, 1947]. Results do not entirely agree with the asperity theory, nor do they in all aspects agree with the cohesive force theory. Maybe with these elastic materials both influences are present, the ratio depending on the conditions of the surfaces. R. G. Boiten, Holland

1182. **Proposed method of test for apparent viscosity of lubricating greases,** *Proc. Amer. Soc. Test. Mat.* **49,** 338–344, 1950.

1183. **Fine, M. E., A simple torsion pendulum for measuring internal friction,** *J. Metals* **188,** 11, sect. 1, p. 1322, Nov. 1950.

Paper is a brief technical note presenting design of a torsional pendulum for measuring internal friction. This particular design, a modification of the pendulum of K_e, is aimed at reducing parasitic or background energy loss to a minimum. Author does not include in his bibliography a recent article by Boulanger [see AMR **3**, Rev. 1102] who developed another solution to the same problem. M. E. Shank, USA

Mechanical Properties of Specific Materials

(See also Revs. 1054, 1055, 1099, 1106, 1110, 1122, 1149, 1155, 1157, 1161, 1166, 1167, 1168, 1173, 1177, 1178, 1179, 1183, 1366)

1184. **Sauer, J. A., and Lemmon, D. C., Effect of steady stress on fatigue behavior of aluminum,** *Trans. Amer. Soc. Metals* **42,** 559–576, 1950.

Paper deals with fatigue behavior of a certain aluminum alloy, Alcoa 14S-T, when it is subjected to the combined action of a steady static stress and a superimposed dynamic stress. Authors consider two types of loading: alternating bending and alternating torsion. While reviewer feels that the experimental results of paper are interesting and suggestive, both the analysis of the data and the design of the experiment (not in engineering but in statistical sense) leave something to be desired. Two specific points could be mentioned:

(a) Suppose several specimens are all run at the *same* mean stress and the *same* alternating dynamic stress; then what sort of variations in cycles to failure are observed? (b) It would be enlightening to put error bands around the curves in figs. 3, 4, 7, and 8. Doing so would bring out the important point that data of the kind treated in paper are subject to a great deal of variability.

It seems to reviewer that it is misleading to give experimental results without stating the magnitude of errors inherent in the data. The design engineer must know the *distribution* of fatigue strengths (where strength has to be suitably defined) under specified conditions before he can intelligently compute factors of safety, determine probable life of a mechanism, etc. Unless this sort of essential information is known, factors of safety could better be called factors of ignorance.

Benjamin Epstein, USA

1185. **Reiner, M., On volume or isotropic flow as exemplified in the creep of concrete,** *Appl. sci. Res.* **A1,** no. 5–6, 475–488, 1949.

Experiments have shown that when concrete creeps under compression, its volume gradually diminishes, the cement flowing into the voids of the concrete. This will generally be the case in porous materials which should show both shear and volume flow, both accompanied by viscous resistances η and ζ of their own. If λ is Trouton's coefficient of viscous traction, it is shown that the shear viscosity and the volume viscosity can be calculated from $\eta = \lambda/2(1 + \sigma_v)$; $\zeta = \lambda/3(1 - 2\sigma_v)$ where σ_v is a "viscous" Poisson ratio relating the rate of contraction e_q to the rate of elongation e_l. This ratio can be calculated from a "deformational" Poisson ratio σ_d by means of $\sigma_v = \sigma_d + \dot{\sigma}_d(d(\ln e_l)/dt)^{-1}$. For a 1:2:4 concrete at an age of two months, calculation shows the following approximate values $\sigma_d = 0.052$, $\sigma_v = 0.075$, $\lambda = 2.4 \times 10^{18}$, $\eta = 10.4 \times 10^{17}$, $\zeta = 9.4 \times 10^{17}$ (the last three in poises). From author's summary

1186. **Loring, S. J., Theory of the mechanical properties of hot plastics,** *Trans. Amer. Soc. mech. Engrs.* **72,** 4, 447–463, May 1950.

Author first derives a general theory of plastic flow on the basis of large strains at a point. He assumes material to be incompressible and to have a generalized isotropy. A generalized solution is developed by means of statistical mechanics. Complete analysis is carried out for two-dimensional strain, and then applied to case of flow in channels and tubes. Theoretical results show good correlation with experimental data.

F. J. Mehringer, USA

1187. **Kawai, Heiji, and Tokita, Noboru, On the Young's moduli and the internal friction of papers,** *J. phys. Soc. Japan* **5,** 5, 365–369, Sept.-Oct. 1950.

Such dynamic properties of paper as modulus of elasticity and damping capacity were investigated, which are of interest to the designer and manufacturer of loudspeakers and acoustical instruments with paper diaphragms. These dynamic properties depend on moisture content of paper and properties of fibers composing the paper. In region of audiofrequencies, they are almost independent of frequency. Above 100 kc, damping capacity increases linearly with frequency. Below 100 kc, the internal friction shifts from this straight line relationship, according to test data presented for various types of papers.

E. George Stern, USA

1188. **Müller, F. H., On the changes of fibers after swelling cycles, IV** (in German), *Koll. Z.* **113,** 2, 91–96, May 1949.

1189. **Wooding, Walter H., Welding air-hardening alloy steels,** *Weld. Res. Suppl.* **15,** 11, 552–564, Nov. 1950.

Investigation of various procedures for the shielded metal-arc welding of an air-hardening alloy steel indicated that the isothermal transformation characteristics of the heat-affected zone considerably influenced the weldability of the material under high restraint. From author's summary

1190. **Shimozuru, Daisuke, Study on the coefficient of internal friction of materials used in civil engineering and architectural structures,** *Bull. Earthq. Res. Inst. Tokyo Univ.* **27,** parts 1–4, 85–89, Jan.-Dec. 1949.

Damping in solid cylindrical specimens of steel, aluminum, ebonite, and wood was measured by decay of free torsional oscillations. Vibrations were initiated by an electromagnet. Specimen, with a disk fixed at each end, was suspended by a fine wire in a

vacuum to reduce external losses. Relative motion of the ends of specimen was optically recorded. Curves given of logarithmic decrement vs. air pressure do not have numerical ordinates, so that effectiveness of the design in reducing external losses cannot be judged. No information as to stresses, physical properties, and condition of the specimens are given, so that results cannot be compared with other work. Several investigators have shown that, at stresses concerned in engineering structures, the decrement is independent of frequency. Reviewer thus believes that curves in present paper showing change in decrement with frequency (also without numerical ordinates) must either have been taken at very low stress levels or may perhaps indicate some difference in specimens used. Different amounts of cold-working in the cold drawn steel wires used in the tests could explain this effect. Donald E. Hudson, USA

1191. Trent, E. M., Carter, A., and Bateman, J., High temperature alloys based on titanium carbide, *Metallurgia* **42,** 250, 111–115, Aug. 1950.

The "hot strength" of cemented carbide is utilized in metal-cutting operations, and investigations of the properties of alloys, based on titanium carbide and containing chronium and nickel or cobalt have shown them to possess many of the properties desirable in gas-turbine materials. The low specific gravity alloys containing a large proportion of titanium carbide make them particularly suitable for turbine rotor blades.

From authors' summary

1192. Krivobok, V. N., and Thomas, R. D., Jr., Impact tests of welded austenitic stainless steels, *Weld. Res. Suppl.* **15,** 9, 493–495, Sept. 1950.

One-half-in. plate samples from a number of steel producers were welded in pairs along one edge, and Charpy keyhole impact specimens cut through the bead. Tests results include as variables the rod composition, testing temperature (room, −105, and − 320 F), and heat treatment. The latter included comparison of the as-welded, 1200 F stress relieved, 1550 F stabilized, and 1950 F annealed, conditions. In addition, impact data are included through the same temperature range for the plate materials employed. No discussion of the results is included.

Charles W. Gadd, USA

1193. Odqvist, F. K. G., and Schaub, C., The yield point of mild steel at nonhomogeneous and compound stress distributions, *Trans. roy. Inst. Technol. Stockholm*, no. 34, 16 pp., 1950.

With mild steels there is an apparent discrepancy between the yield stress in simple tension or compression and that obtained from nonuniformly stressed specimens (e.g., beams, thick-walled tubes, torsion specimens, etc.) as the maximum calculated elastic stress at the instant when Lüders' lines first appear. Authors explain this by the hypothesis (extension of a theory presented by Nakanishi, 1931) that Lüders' lines first appear some time after yield stress has been reached and, in fact, only after a certain portion of the most highly strained section of specimen has yielded. Material that yields suffers a loss of strength proportional to (more generally, a function of) the amount by which its strain exceeds the strain at the yield point. In case of bending, for example, at any instant after yielding begins but before Lüders' lines appear, the stress increases linearly with distance from neutral axis, reaching a maximum (the lower yield stress), and then decreases as outer fibers are approached (in agreement with x-ray measurements by Bollenrath and Schiedt in 1938, which were repeated by authors). As amount of bending increases and yielded region spreads, the resisting moment increases at first. It reaches a maximum eventually. According to authors, this maximum occurs at the instant when Lüders' lines appear. Authors apply this hypothesis to beams and to thick-walled tubes with internal pressure. Experiments were made which lend support to the hypothesis. M. P. White, USA

1194. Norris, Charles B., Strength of orthotropic materials subjected to combined stresses, *For. Prod. Lab. Rep.* no. 1816, 19 pp., July 1950.

A theory of the resistance of orthotropic materials to stresses applied at angles to the mutually perpendicular natural axes and to combined stresses with respect to these axes is developed from the Hencky-von Mises theory of energy due to change in shape. Formula is based on an assumed orthotropic material made by introducing uniform, regularly spaced, prismatic voids into an isotropic material and is shown to be without serious error when applied to an aeolotropic material containing irregularly shaped and spaced voids. General equations are derived which describe the limiting stresses of cellular orthotropic materials as surfaces of three ellipsoids with a common center. The formula, reduced to apply to macroscopically isotropic materials, is shown to yield the currently used diagram of limiting stresses in metals. Specific equations for the strength of an orthotropic material subjected to a single stress at angles to natural axes are derived and checked by tests of Douglas fir, plywood, and a glass fabric laminate. Experimental and theoretical values agree favorably.

Stephen B. Preston, USA

1195. Ellis, O. B., Effect of weather on the initial corrosion rate of sheet zinc, *Proc. Amer. Soc. Test. Mat.* **49,** 152–170, 1949.

Weight losses of 26 samples (5 specimens each) of sheet zinc exposed to a mild industrial atmosphere for 28-day periods are studied with relation to weather conditions. Weight losses are found to be especially dependent on what happens during the initial period of exposure, i.e., when the initial (5-day) rate of loss is high, the rate of loss remains high (at least up to 12 months) although smaller than initially. Increased periods of rainfall or high humidity cause increased rates of weight loss. The initial rate of corrosion of low-copper iron did not appear to vary to the same extent with weather conditions. M. P. White, USA

1196. Sinclair, David, A bending method for measurement of the tensile strength and Young's modulus of glass fibers, *J. appl. Phys.* **21,** 5, 380–386, May 1950.

Tensile strength of glass fibers is known to vary greatly depending principally upon diameter and length of tested fiber. Author describes a new method of testing: a small loop is twisted in a fiber 1- or 2-in. long and the ends are pulled until fiber breaks. The looped fiber is equivalent to a very short straight fiber, since break must occur at the point of minimum radius of curvature; the strain can be readily measured. The instrument, a chainomatic balance, is described. Results of tests show that tensile strengths were two to three times higher than that of the strength fiber method. The equation of the loop is calculated and is found to fit closely to the photographed form of the loop. Formulas for calculating the minimum radius of curvature, tensile strength, and Young's modulus are produced.

D. DeMeulemeester, Belgium

1197. De Sy, A., Production and properties of nodular cast iron (in Dutch), *Ingenieur* **62,** 29, 61–68, July 1950.

In nodular cast iron C is present as spherical particles, while in normal cast iron it is present as flakes. The strength and ductility is therefore much better. It is obtained by adding elements like Ce, Si, or Mg to the melt in a greater amount than is necessary for the combination with S and O. The noduli start from a

nucleus of Si-ferrite and contain more Mg than the matrix. Instead of Ni-Mg- or Cu-Mg-alloy (USA practice), Si-Mg-alloy is preferred for Mg-addition. The mechanical properties (tens. str. 40-120, compr. str. 80-120. Br. hardn. 140-420. El. mod. 16000-18000 kg/mm²) vary between those of cast iron, malleable cast iron and cast steel, depending on the composition (preferable 3.2-3.7% C, 2.2-2.5 Si), the casting conditions, and heat treatment.
J. H. Palm, Holland

1198. **Hopkin, L. M. T., A simple constant stress apparatus for creep testing,** *Proc. phys. Soc. Sec. B*, **63,** part 5, 365 B, 346–349, May 1950.

On the assumption of constant volume of specimen in creep, a cam-moment-arm loading device is designed so that as cross-sectional area of the specimen decreases, the applied load decreases, maintaining a constant stress on specimen. Novel use of elastic straps replaces pivot fulcrum. The design of the constant stress apparatus is simple and economical. For strains up to 15%, load error does not exceed 0.3%; for strains up to 100%, load error does not exceed 0.8%. Position of the center of gravity of the cam-moment-arm is considered as the major source of error. Creep data obtained in two tests indicate an excellent agreement with the Andrade equation for creep under constant stress [Andrade, E.N. da C., *Proc. roy. Soc. Lond. Ser. A*, **84**, p. 1, 1910].
George H. Lee, USA

1199. **Oberg, T. T., and Rooney, R. J., Fatigue characteristics of aluminum alloy 75S-T6 plate in reversed bending as affected by type of machine and specimen,** *Proc. Amer. Soc. Test. Mat.* **49,** 804–814. 1949.

Vibratory nonrotating cantilever fatigue tests were made in a Krouse plate fatigue machine (fixed deflection), using test specimens of round, square, and rectangular critical sections. Rotating-beam specimens (round) were tested in an R. R. Moore type rotating-beam (constant load) fatigue-testing machine. All specimens were machined to a critical thickness of diameter of ½ in. from a single ³/₈-in. plate of aluminum alloy 75S-T6 with 5.1 to 6.1% Zn, 1.2 to 2.0% Cu, 2.1 to 2.9% Mg, and a tensile strength of 84,200 psi in longitudinal direction. Fatigue properties in reversed bending for various types of test specimens (cantilever round section, simple rotating beam, cantilever square section, cantilever constant strength, cantilever rectangular section) are reported.

Nonrotating round specimens had the least surface area at region of maximum stress and developed the highest fatigue value. However, an increase in this critical area, as in case of square and rectangular specimens, was associated with a decrease in fatigue strength. It is believed that variations in fatigue strengths for the various types of specimens used can be attributed solely to shape of critical section, although methods of finishing may have had a very slight effect. Annealed 18% chromium, 8% nickel steel, which has very low notch sensitivity, developed same fatigue limit for R. R. Moore rotating-beam specimens as that for rectangular cantilever-sheet specimens.
E. Siebel, Germany

1200. **Valore, Rudolph C., Jr., Bowling, James E., and Blaine, R. L., The direct and continuous measurement of bleeding in Portland cement-water mixtures,** *Proc. Amer. Soc. Test. Mat.* **49,** 891–908, 1949.

An apparatus is described for measuring continuously the water of bleeding as it separates from freshly mixed Portland-cement water mixtures. Bleeding water is collected by flotation through carbon tetrachloride. Tests of neat cement pastes at various water-cement ratios by carbon tetrachloride method and by float-subsidence method (in which bleeding is identified as the subsidence of the paste surface) yielded bleeding-time curves of similar types, but bleeding rates and capacities calculated from carbon-tetrachloride test data averaged about 80% of those obtained from subsidence data. Carbon-tetrachloride method was found to give results of satisfactory reproducibility. Experiments for evaluating container boundary-friction effects and examples of channeled bleeding have been described. Results of the application of carbon-tetrachloride method to concretes are also presented.
From authors' summary

1201. **Miklowitz, Julius, The effects of temperature and material structure on the fracture properties of medium-carbon steel,** *Proc. Amer. Soc. Test. Mat.* **49,** 602–617, 1949.

Results of tension and V-notch Charpy tests in the temperature range −65 to +25 C of two medium-carbon (0.25% C) open-hearth steels, one deoxidized with silicon, the other with silicon and aluminum, in several conditions of carbide dispersion (fine pearlite, coarse pearlite, spheroidized). Each microstructure was developed in both fine and coarse austenite grains in both steels.

Effects of carbide dispersion and austenite grain size are shown in plots of conventional strain at ultimate load and at fracture, of true stress at ultimate load, of average true stress at fracture, and of notch-impact energy against temperature.
George V. Smith, USA

Mechanics of Forming and Cutting

1202. **Shaw, Milton C., and Strang, Charles D., Jr., Metal transfer in the cutting process,** *J. appl. Phys.* **21,** 4, 349–350, Apr. 1950.

Radioactive SAE 4027 steel was machined with a face milling cutter to demonstrate that even in presence of carbon tetrachloride as cutting fluid, invisible particles of work material are welded to the tool face. This deposit of metal showed up on an x-ray film when placed in contact with the cutter for a period of two weeks. Method should be useful in determining the effectiveness of cutting fluids in preventing metal pickup as well as in detecting small rates of tool wear.
E. G. Thomsen, USA

1203. **Nancarrow, H. A., and Atkinson, E. B., The extrusion of thermoplastics,** *Some rec. develop. Rheol.*, United Trade Press, London, pp. 1–13, July 1950.

This brief paper discusses conditions under which thermoplastic materials can be satisfactorily extruded, specifically in screw-type extruding machines, and describes how information useful in predicting the relation between delivery and pressure can be secured from small scale experiments. A simple plastometer is described, consisting essentially of a heated cylinder and a ram forcing a predetermined amount of plastic through a die, with provisions for load, speed, and temperature measurement. General relations are analytically derived between flow rate and pressure gradient for solid and hollow circular cylindrical extrusions. Extrusion swell or recovery is briefly touched upon; desirability is pointed out of keeping ratio of apparent viscosity to elasticity modulus low, to prevent distortion, roughness, and swelling.
A. R. C. Markl, USA

1204. **Stremsdoerfer, J., Contribution to the study of rolling pressures** (in French), *Rev. Métall.* **47,** 4, 237–259, Apr. 1950.

Improved way of calculating the force and moment on the rolls for hot rolling. The underlying hypotheses are: (a) Work done to attain a certain plastic deformation is independent of the way in which this deformation is obtained; (b) critical shearing stress

for plastic deformation is constant; (c) radial pressure at contact between roll and material is constant; (d) resultant force on roll applies in middle of contact arc; (e) no lateral deformation. The formulas obtained are compared with those of Ekelund and appear to fit the measured values somewhat better. The final formulas are: For a bloom: force = $2\omega bR^{1/2}(h_1 - h_2)^{1/2}$; moment of couple: $2\omega bR(h_1 - h_2)$. For sheet: force = $2\omega bR^{1/2}(h_1 - h_2)^{1/2}$ $[1 + {}^1/_2\, R^{1/2}(h_1 - h_2)^{1/2}/(h_1 + h_2) + (h_1 - h_2)/(h_1 + h_2)]$; moment of couple: $2\omega bR(h_1 - h_2)$ $[1 + {}^1/_2\, R^{1/2}(h_1 - h_2)^{1/2}/(h_1 + h_2) + (h_1 - h_2)/(h_1 + h_2)]$.

Here R is radius of rolls; b breadth; h_1 and h_2 heights of specimen before and after the rolls; ω radius of Mohr circle as previously used by Ekelund: $\omega = C_1 + C_2V(h_1 + h_2)^{1/2}R^{-1/2}/(h_1 - h_2)$; C_1 and C_2 specific properties for the material; V tangential velocity of rolls. C. Zwikker, Netherlands

1205. Roberts, C. W., and Walters, B., The rolling of zinc and zinc-rich alloys, *J. Inst. Met.* **76,** part 5, 557–580, 1950.

Casting of zinc sheet and strips by means of the open top horizontal and vertical mould, as well as the vacuum casting process are described. A particularly successful way to overcome the columnar crystalline structure which makes the casting weak is by addition of titanium. Pack and strip rolling process, as well as the less familiar Hazelett and electrolytic processes, are also described. Rolling is then considered from a metallurgical viewpoint.

Zinc recrystallizes at normal cold rolling temperatures. Unlike most metals, it cannot be hardened by cold working. It readily deforms under slowly applied loads but its resistance to deformation increases rapidly with increased rate of loading. After hot rolling, ductility is greater in direction of rolling, while ultimate strength is slightly higher in transverse direction.

Zinc-rich alloys are also considered from fabricational and metallurgical viewpoints. Bernard W. Shaffer, USA

1206. Bodart, Émile, Study of the properties of surfaces (in French), Centennial Iss., Faculty of Science, Univ. Liége, 168–174, 1947.

Hydraulics; Cavitation; Transport

(See also Revs. 1221, 1419)

1207. Rouse, Hunter, Engineering hydraulics. Proceedings of the fourth hydraulics conference Iowa Institute of Hydraulic Research, New York: John Wiley & Sons, Inc.; London; Chapman & Hall, Ltd., 1950, xiii + 1039 pp. $15.

Book contains a comprehensive review of principles and applications of mechanics of incompressible fluids as developed in the last fifty years. In the first chapter Dr. Rouse, director of the Institute, reviews the fundamental principles of flow. Much of this material was already available in his "Fluid mechanics for hydraulic engineers," but this condensed presentation serves as an introduction to the volume, stressing definitions, properties, methods of analysis, and fundamental equations.

Second chapter, prepared by the late G. E. Warnock, perfects this purpose by discussing basic laws of hydraulic similitude and their application to model studies. Practical information is made available and several examples are given such as flow in closed conduits, including cavitation, flow in open channels, tidal and wave models, and hydraulic machinery.

In chapter 3, J. W. Howe presents various methods of measurement: Measurement of fundamental quantities (distances, elevations, times, forces), of derived quantities (pressures, velocities), and discharge measurements in closed conduits and in open channels.

Hydrology is the subject of chapter 4, written by G. R. Williams. Emphasis is laid on the principles of this science, but numerous applications are given. Meteorological principles, climatology, runoff, ground water, storm rainfall, storm runoff, relation between storm rainfall and storm runoff, theory of surface runoff, and drainage design are the various topics studied.

In chapter 5 (flow of ground water) C. E. Jacob, after deriving the fundamental equations based on a generalization of Darcy's law, studies their application to the problem of confined flow, both steady and unsteady. Numerous problems on well systems are treated.

Steady flow in pipes and conduits is analyzed by V. L. Streeter in chapter 6. Modern concepts on velocity distribution and energy loss are reviewed. Information is given on surface and form resistance in various cases and compound and complex systems of pipe lines are studied, including a solution of pipe network problems by the Hardy Cross method.

Unsteady-flow problems in closed conduits are treated in chapter 7. J. S. MacNown discusses water hammer in simple and complex conduits and analyzes the function of surge tanks. Effect of pipe resistance is taken into consideration, and attention is given to hydraulic design and to consequences of water hammer in pumping systems.

Chapter 8, by A. T. Ippen, contains considerations on the design and operation of transitions and controls for open channel flow in the subcritical and supercritical flow stages. This is probably one of the newest and lesser known aspects of engineering hydraulics. This chapter also contains information on stilling basins.

In chapter 9, C. J. Posey reviews the problem of gradually varied flow in open channels, giving various methods used to compute and plot backwater curves both in uniform and non-uniform channels. This chapter also includes a study of the hydraulic jump.

The complex problem of flood routing is analyzed mathematically in chapter 10 by B. R. Gilcrest. This is followed by studies of routing of floods through reservoirs and through open channels.

G. H. Keulegan is the author of chapter 11 on wave motion. Shallow-water waves, deep-water waves, oscillatory waves in shallow water, transformation of waves, open channel surges, and internal waves are topics included.

Chapter 12, prepared by C. B. Brown, deals with the mechanics of sediment transportation. After first reviewing sediment properties, it is indicated how the information obtained is used to design canals and desilting works, as well as in the stabilization and improvement of rivers. Other problems include sedimentation in reservoirs and coastal sediment problems.

The last chapter, on hydraulic machinery, was prepared by James W. Daily. The theory of hydraulic machinery, based on methods of modern fluid mechanics is explained with application to centrifugal and axial-flow pumps, propulsion machinery, hydraulic turbines, hydrodynamic transmissions, and positive displacement machinery.

An appendix includes: (a) A list of symbols which will be very useful in uniformizing notations; (b) dimensional analysis considerations; and (c) information on properties of common fluids.

This volume, the only one of its kind in the English language, has been prepared with extreme care. Each chapter has been written by one of the foremost specialists in the field. Rather than present new information, a successful attempt has been made to prepare a comprehensive synopsis of methods involved and to discuss typical applications. Although not a textbook, "Engineering hydraulics" will undoubtedly be used as a reference in many graduate courses. Although not a handbook, it

contains a wealth of information for the practicing engineer. Primarily, however, it constitutes a true encyclopedia of modern hydraulics. André L. Jorissen, USA

1208. **Marchetti, Mario, Pressure losses in flexible corrugated and smooth pipes** (in Italian), *Energia Elett.* **27,** 1, 11–22, Jan. 1950.

First part of paper summarizes measurements made in the United States (Freeman and Yarnell) and by Gibson in Great Britain. Author tested several flexible rubber pipes, one being smooth inside (40-mm diam) three others being corrugated (70-mm diam), and reinforced by a spiral with height varying from one case to another. Tests were made with straight and with bent flexible pipes. Results are discussed and represented by diagrams as a function of Reynolds number and compared with the formula given by Freeman. Manning-type formulas are worked out for the different types of flexible pipes. Charles Jaeger, England

1209. **Jerie, Jan, Flow through straight-through labyrinth seals,** *Proc. seventh int. Congr. appl. Mech.* **2,** part 1, 70–82, 1948.

1210. **Silber, Robert, On the slope of the free surface in the neighborhood of the critical depth in open-channel flow** (in French), *C. R. Acad. Sci. Paris* **230,** 1/2, 1450–1452, Apr. 1950.

Derivation of the customary equation of gradually varied open-channel flow involves assumption that effects of vertical acceleration are negligible. Only in vicinity of the critical depth is this assumption not justified, and the slope of the computed surface curves then approaches the physically possible limit of finity. Author attempts to derive an expression which is in closer agreement with reality by taking into account the vertical component of velocity, but result is still merely an approximation because he tacitly assumes pressure distribution to remain hydrostatic.

Hunter Rouse, USA

1211. **Gentilini, Bruno, Action of a lateral weir on the positive ascendant wave in a canal** (in Italian), *Energia Elett.* **27,** 1, 1–10, Jan. 1950.

H. Favre, G. de Marchi, and others have studied experimentally or theoretically the lateral weir on a canal under steady-flow conditions. Author made a series of tests on a lateral weir working under unsteady-flow conditions by producing a positive translation wave. Canal width was 25 cm, length of side weir was variable from 80 to 200 cm; water depth, identical to the side weir height, was 15 cm. Waves, produced in the canal by sudden closing of a gate at the downstream end, were recorded by a cine-camera taking 24 frames per sec. Results agreed with theories given by Drioli and Citrini. The wave velocity was checked against the theoretical values, and the maximum wave height h^* was compared with the average wave height h. It was found that the lateral weir has an effect of amortization on the wave, which can be observed upstream and downstream of the weir.

It was found that two lateral spillways arranged symmetrically on both sides of the canal will give the same amortization of wave height as one single lateral spillway of same total crest length.

Charles Jaeger, England

1212. **Escande, Leopold, Experiments on the operation of hinged spillway crests** (in French), *C. R. Acad. Sci. Paris* **230,** 14, 1341–1342, Apr. 1950.

Model tests at a 1:8 scale on a series of movable spillway crests are described in very brief detail. The crest form was based on the Creager profile for a 1-m head, with a small circular arc at the upstream pivot point, and the crests varied in length from 1.4 to 3 m. Plotted results include a typical flow net, a comparison of computed and measured pressure distributions along the profile, and values of torque about the pivot point as a function of head, crest length, and (indirectly) crest inclination.

Hunter Rouse, USA

1213. **Rousselet, Réné, and Schlag, Alb., Contribution to the normalisation of classic Venturi-tubes,** *Rev. Univ. Min.* (9), **6,** 9, 301–308, Sept. 1950.

British and French standards for Venturis are analyzed briefly. Results of hydraulic tests, recently executed in the Hydraulic Laboratory at the University of Liége, are published. Venturis with contraction-ratios from 0.10 up to 0.65, connected upstream to rough or smooth straight pipes of 100-mm diam were tested. Effects of Reynolds number and pipe roughness are examined. Experiments indicate values for discharge coefficients situated between those given by the British and French standards.

E. Mühlemann, Switzerland

1214. **Leniger, H. A., and Jansen, J. R., Fluidization** (in Dutch), *Ingenieur* **61, 62,** 49, 7, 11, p. Ch. 47–50, 11–18, 23–27, Dec. 1949, Feb., Mar. 1950.

This is a timely and comprehensive review of mainly experimental results on fluidization problems carried out in the United States since 1943 and previously published as a result of a symposium on dynamics of fluid-solid systems [*Ind. Eng. Chem.* **41,** June 1949].

Paper takes up the following topics: Nomenclature of recommended terms, characteristics of fluidized beds, pressure drop-fluid velocity relations, gas and solid mixing in fluidized beds, heat- and dust-transfer characteristics.

G. A. T. Heyndrickx, Belgium

1215. **Evangelisti, Giuseppe, Piezometric wells and stability of regulation** (in Italian), *Energia Elett.* **217,** 5, 6, 253–269, 353–367, May, June 1950.

Investigation of a hydroelectric construction consisting of a great basin, a communication tunnel (designated *galleria*) which runs into a vertical shaft with a free surface. Then follows a second tunnel under high pressure leading over a distributor to the water turbine which is equipped with an automatic regulator for governing the opening of the distributor to maintain the equality between the driving force and the force resistant. Up to the formulation of the equations of the problem under the classical assumptions, author gives the complete solution and discusses particularly the question of the transfer from one stationary state of motion to another. The function of the direct and indirect regulation under different circumstances is given, as well as numerical applications and a complete bibliography.

Th. Pöschl, Germany

1216. **De Marchi, Giulio, On the change of the regime of a linear flow with free surface in a river bed of constant cross section** (in Italian), *Energia Elett.* **27,** 3, 125–132, Mar. 1950.

Analysis of stationary flow in an open channel, with constant cross section of arbitrary shape, near the transition through critical depth. Necessary conditions are specified quantitatively in order that the flow can be treated as gradually variable. Situations are pointed out which allow the gradual transition from subcritical to supercritical flow or vice versa, and, for the different situations, the values of bottom and surface slopes in the control section are determined, where the distance of the energy line from the bottom is a minimum. Detailed examination of cases with increasing or decreasing downstream bottom slope; last

case is normally verified by natural streams. If bottom curvature is small (limiting value is given), it is possible, by decreasing slope, to gradually change from supercritical to subcritical flow (without jump), which is verified by natural streams.

Duilio Citrini, Italy

Incompressible Flow: Laminar; Viscous

(See also Revs. 1031, 1160, 1210, 1278, 1293, 1319, 1321, 1387, 1402, 1411)

1217. Goldstein, S., and Lighthill, M. J., A note on the hodograph transformation for the two-dimensional vortex flow of an incompressible fluid, *Quart. J. Mech. appl. Math.* **3,** part 3, 297–302, Sept. 1950.

It is pointed out that branch lines occur in the hodograph transformation of two-dimensional incompressible vortex flow. Along these lines the hodograph plane folds back in the same way as in supersonic potential flow of a gas. As an example, the hodograph plane corresponding to a flow of uniform shear past a cylinder is investigated. Even in this simple case there are two branch lines along which six distinct sheets of the hodograph surface are connected. L. J. F. Broer, Holland

1218. Allen, H. Julian, Pressure distribution and some effects of viscosity on slender inclined bodies of revolution, *Nat. adv. Comm. Aero. tech. Note* 2044, 19 pp., Mar. 1950.

Munk's airship theory is used to derive a simple approximate equation for the increment in pressure distribution associated with the incidence angle of bodies of revolution. Calculated and measured pressure distributions are in good agreement over the forebody but not over the afterbody. It is pointed out that the differences between theory and experiment are indicative of the existence of local regions of separated flow, occurrence of which is in accord with oblique viscous flow considerations.

Gerald E. Nitzberg, USA

1219. Sears, W. R., Potential flow around a rotating cylindrical blade, *J. aero. Sci.* **17,** 3, 183–189, Mar. 1950.

An infinitely long cylinder, of arbitrary section, is considered to rotate uniformly about an axis normal to it, in an incompressible inviscid fluid "otherwise at rest." Velocity field is determined simply in terms of two-dimensional potential of flow of a uniform stream normal to cylinder. A special case is considered. Trailing vortexes are absent. M. J. Lighthill, USA

1220. Funaioli, E., On airfoil cascades with fixed characteristics (in Italian), *Aerotecnica* **30,** 3, 114–119, June 1950.

Author describes a method of conformal transformation for the calculation of airfoil cascades with prefixed characteristics. He starts from the flow around a circle outside of which an isolated singularity is assumed which must be suitably completed inside the circle. The conformal transformation is effectuated in two steps. In the first, the circle is transformed into a logarithmic spiral segment, and another circle enclosing and touching the fundamental circle into a single profile with the logarithmic spiral segment as skeleton. A second conformal transformation of logarithmic type finally gives a profile cascade with finite thickness but zero angle of the trailing edge. W. Wuest, Germany

1221. Powell, Ralph W., Resistance to flow in rough channels, *Trans. Amer. geophys. Un.* **31,** 4, 575–582, Aug. 1950.

Paper presents, essentially, a second interim report of the author's search for a better equation to replace that of Manning. Results of his previous paper are modified somewhat but, in reviewer's opinion, are still not conclusively better than those of Manning. Further evidence is needed, some of which could come from a study proposed in the paper.

D. F. Gunder, USA

1222. Rumer, Yu. B., Annular turbulent source (in Russian), *Dokladi* **64,** 4, 463–466, Feb. 1949.

Consider a free jet flowing from a narrow annular slit of radius R and breadth $2h$ cut in a cylindrical surface. Take a cylindrical coordinate system with origin at center and with z-axis along axis of the annulus. Use the coordinates r, $\varphi = z/ar$, where a is a dimensionless structure constant, characterizing the turbulence. Assume that radial velocity u is of the form $u(r, z) = u_m(r)f(\varphi)$, where $0 < r < \infty$, $0 < \varphi < \varphi_e$, φ_e denoting the value of φ at the edge of jet. Then, for radial and axial velocity, following expressions are obtained: $u = u_m F'(\varphi)$, $v = au_m[F'(\varphi\varphi - F(\varphi)$, where $u_m = u_0R_0/r$, $R_0 = 0.685$, $F'(\varphi) = f(\varphi)$, u_0 denotes the initial velocity and $F(\varphi)$ satisfies the equation $c^2F''^2 - a^2FF''' = 0$, which for $a^3 = c^2$ coincides with Tollmien's equation in the plane case. Results of paper may be summarized as follows:

1. Distribution of radial velocity in plane of symmetry of jet is the same as that of axial velocity in the case of a circular jet, i.e., velocity decreases inversely proportional to distance from pole.

2. Distribution of radial velocity with respect to φ is identical with distribution of longitudinal velocity with respect to φ in a plane jet.

3. Distribution of axial velocity with respect to φ differs from that of transverse velocity in the plane case. While in latter case $v = au_m(F'\varphi - F\ 2)$, in problem of annular source $v = au_m(F'\varphi - F)$.

4. The structure constant a is related to the constant c in plane case by $a^3 = 2c^2$, while in case under consideration $a^3 = c^2$.

A table giving numerical values of F, F', F'' and $F'\varphi - F$ for the range of φ between 0 and $\varphi_e = 4\pi/3(3)^{1/2} \approx 2.4$ is added.

E. Leimanis, Canada

1223. Heertjes, P. M., Industrial filtration, *Research* **3,** 6, 254–259, June 1950.

Author discusses present theoretical knowledge of the filtration process. Complexity of the problem is emphasized and the attempts to differentiate between various factors determining the filtration rate are given. Paper contains an extensive bibliography. R. C. Roberts, USA

1224. Vladimirsky, Serge, On the nonstationary motion of two plates (in French), *C. R. Acad. Sci. Paris* **230,** 22, 1928–1930, May 1950.

Author considers the disturbance in a uniform two-dimensional stream due to general, unsteady, small motion of two flat plates, placed edgewise to stream, one behind the other. Disturbance potential involves two unknown functions, defining strength of the vortex distribution in the wake behind each plate. These functions are to be determined by two integral equations, which are stated but not solved. Maurice Holt, England

1225. Vladimirsky, Serge, Differential non-uniform motion of two plates (in French), *C. R. Acad. Sci. Paris* **231,** 3, 211–213, July 1950.

This follows an earlier paper (see preceding review) concerning small, two-dimensional unsteady motions due to two flat plates placed one behind the other, initially edgewise to the main stream. In former paper, both plates executed the same motion in each

case; in present paper differential movements are allowed; e.g., one of the plates may be kept fixed. Maurice Holt, England

1226. **Lewis, D. J., The instability of liquid surfaces when accelerated in direction perpendicular to their planes. II,** *Proc. roy. Soc. Lond. Ser. A*, **202,** 1068, 81–96, June 1950.

An apparatus for accelerating small quantities of various liquids vertically downwards at accelerations of the order of $50g$ ($g = 32.2$ ft/sec^2) is described, and the behavior of small wavelike corrugations initially imposed on the upper liquid surface has been observed by high-speed shadow photography. Instability observed under a wide variety of conditions has been analyzed, and the initial phases have been found to agree well with the first-order theory given in part I [Taylor, G., title source, **201,** 192–196, 1950; AMR 4, Rev. 757]. When disturbance has attained a considerable amplitude, the first-order equations cease to apply and it changes from a wave into large round-ended columns of air extending into liquid. The air columns attain a steady velocity relative to accelerating liquid. Main body of liquid below them is accelerated as though they did not exist.

From author's summary by H. Görtler, Germany

1227. **Jones, Robert T., Leading-edge singularities in thin-airfoil theory,** *J. aero. Sci.* **17,** 5, 307–310, May 1950.

Thin airfoil theory gives a pressure singularity at the leading edge, which leads to an error in the drag. Author obtains the correction due to this singularity and shows that, in case of the wing in incompressible flow, it exactly balances the predicted negative drag. He then considers the thin elliptic cone in supersonic flow (previously treated by Squire) and finds that the leading-edge correction increases the drag found by linearized theory. The extension to nonconical fields of the method of finding this correction is described. Maurice Holt, England

1228. **Paron, André, Minimum radius of curvature of the leading edge of a wing profile** (in French), *C. R. Acad. Sci. Paris* **230,** 22, 1931–1932, May 1950.

The minimum radius is computed in terms of corresponding radius of curvature of curve C_1, which maps into the airfoil profile, and of two constants occurring in the approximation to the mapping function in neighborhood of the forward branch point.

M. V. Morkovin, USA

1229. **Moriya, Tomijiro, On Blasius formulae referred to moving axes,** *J. Soc. appl. Mech. Japan*, 81–85, 1950.

Forces and moments are computed on a two-dimensional cylinder of arbitrary cross section moving with steady translational and pitching velocities. The results are referred to axes fixed to the cylinder. Author concludes that use of these axes is particularly convenient when the cylinder has both translational and angular velocities, or an angular velocity in a special flow such as in a centrifugal pump. Several examples are given to illustrate validity of the theoretical work. R. E. Wilson, USA

1230. **Iberall, Arthur S., Attenuation of oscillatory pressures in instrument lines,** *J. Res. nat. Bur. Stands.* **45,** no. 1, 85–108 = *Trans. Amer. Soc. mech. Engrs.* **72,** 5, 689–695, July 1950.

A thorough analysis for the attenuation and lag of an oscillating pressure applied to one end of a fluid conduit with a pressure recording instrument at the other. An elementary theory is presented in which a viscous incompressible fluid is assumed to move according to Poiseuille's law of resistance. The calculations produce a time constant associated with dimensions of the tube, internal volume of the end device, and average state of the gas in the tube. An attenuation factor is also determined from which the attenuation and phase lag may be calculated.

The elementary theory does not include all first-order effects, and author takes these into account by a relaxation of the more restrictive assumptions involved. Following factors are taken into account: Compressibility, finite excess of pressure, fluid inertia, finite length of tube, and heat conduction. Method for computing transmitted pressure is illustrated by examples.

This subject is important to the experimentalist and the theory should be tested in the laboratory. In particular, further attention should be given to the question of the exponent in the polytropic process assumed to occur in the tube.

G. N. Patterson, Canada

1231. **Imai, I., On the velocity of falling raindrops** (in Japanese), *J. meteor. Soc. Jap.* **28,** 4, 113–118, Apr. 1950.

Author shows theoretically that, if flow around the drop is a potential one and deformation is small, the shape of a falling raindrop is an oblate spheroid whose axis is vertical. He then computes the fall velocity on the assumption that the increase in cross area due to deformation only causes a decrease in fall velocity, and that drag coefficient of an oblate spheroid with small eccentricity is approximately equal to that of a sphere whose radius is equal to major radius of the former. The computed values of fall velocity agree very well with recent measurements of R. Gunn and J. D. Kinzer in range of diameters less than 2 mm. [This paper appears in English in the *Geophysical Magazine* **21,** 3, Tokyo.] From author's summary

1232. **Johnson, John C., Measurement of the surface temperature of evaporating water drops,** *J. appl. Phys.* 21, 1, 22–23, Jan. 1950.

Measurements made of surface temperature of evaporating water drops are in agreement with requirements of theory enunciated by Fuchs, which states that the temperature is determined by the difference between the ambient and saturated vapor density of the drop. From author's summary

1233. **Binnie, A. M., and Harris, D. P., The application of boundary-layer theory to swirling liquid flow through a nozzle,** *Quart. J. Mech. appl. Math.* **3,** part 1, 89–106, Mar. 1950.

Authors consider flow of incompressible fluid into a converging-diverging nozzle with axial symmetry. Convergence is assumed gradual and radial velocities are neglected. Axial combined with rotary (swirl) motion is specified. Velocity and pressure gradients computed for an irrotational flow give conditions at outer seam of boundary layer, and von Kármán momentum-integral equation is written for boundary-layer flow. With assumed velocity distributions (after Pohlhausen), equation is solved numerically for a particular case. Authors conclude boundary-layer effects are small for water. Surface-tension effects for partially filled nozzle with axial air core are also estimated and found negligible. E. D. Kane, USA

1234. **Taylor, G. I., The boundary layer in the converging nozzle of a swirl atomizer,** *Quart. J. Mech. appl. Math.* **3,** part 2, 129–139, June 1950.

The boundary layer along a conical surface immersed in a liquid swirling about the cone axis is studied. Flow inside boundary layer is assumed to be axially symmetrical with three velocity components. The simplified boundary-layer equations are then solved approximately by Pohlhausen's method. Improved solution to this problem was given later by A. M. Binnie and D. P. Harris.

Courtesy of Mathematical Reviews Y. H. Kuo, USA

1235. Manwell, A. R., A method of variation for flow problems, I, *Quart. J. Math. Oxford Ser.* **20,** 166–189, 1949.

Problems considered here require determination of boundary conditions (e.g., profile of an airfoil) to minimize a functional of both boundary and velocities in two-dimensional incompressible potential flow about it. To begin with, author finds profile for which $IA^{-1/2}$ is least, I being $_c\int vds$, v the flow speed at contour C, and A the area enclosed by profile. Method used, which is quite general, is similar to that employed in an earlier paper [*Quart. J. Mech. appl. Math.* **1**, 365–375, 1948; AMR **2**, Rev. 326]. A more difficult case is the minimizing of $A^{-1/2}{}_c\int v^2dx$; this is considered, and also a problem solved by Pólya [*Proc. nat. Acad. Sci. Wash.* **33**, 218–221, 1947]. The general principle deduced is that extremum occurs when the functional is stationary for all variations in which the physical dimensions and velocities are both changed infinitesimally. In some problems it is convenient to use the hodograph plane. It is shown that variations of type of functionals considered here are independent of shape of local variations of boundary conditions in this plane; this is in agreement with principle stated above. An example is worked out, which includes the determination of δA for a variation in hodograph plane. Finally, application of this method to subsonic compressible flow is discussed. W. R. Sears, USA

1236. Manarini, Mario, On the paradoxes of D'Alembert and of Brillouin in fluid dynamics (in Italian), *Boll. Un. Mat. Ital.* (3) **4,** 352–353, 1949.

Author replaces the condition $vr^3 = 0$ in mean at infinity by the less restrictive condition $vr^2 = 0$ in mean at infinity in direction of axis of motion in his extension of Brillouin's paradox.
L. M. Milne-Thomson, England

Compressible Flow, Gas Dynamics

(See also Revs. 997, 1281, 1285, 1288, 1310, 1315, 1316, 1317, 1318, 1335, 1380)

1237. Ferrari, Carlo, On the determination of some fields of supersonic flow (in Italian), *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat.* **7,** 6, 277–283, Dec. 1949.

Author shows how some problems of linearized supersonic three-dimensional flow, which are ordinarily treated by the superposition of fundamental solutions, can be solved by the superposition of particular solutions which are periodic in one direction normal to the free-stream direction. The applications given can be found in *J. aero. Sci.*, Sept. 1949. G. Guderley, USA

1238. Stewartson, K., Supersonic flow over an inclined wing of zero aspect ratio, *Proc. Camb. phil. Soc.* **46,** part 2, 307–315, Apr. 1950.

The linearized theory of compressible flow is used to calculate the lift at supersonic speed of a thin, inclined wing of rectangular plan form and vanishing aspect ratio. By means of the Laplace transform, an asymptotic solution is obtained which describes the chordwise lift distribution at large distances from the leading edge. This solution is compared at a free-stream Mach number of $2^{1/2}$ with Gunn's rigorous solution for the lift along center line of a wing of small but finite aspect ratio [*Phil. Trans. roy. Soc. Lond.* **240,** 327–373, 1948; AMR **2,** Rev. 898]. Results of the two calculations are virtually indistinguishable in the region between 4 and 5 semispans from leading edge, the latter station being the practical downstream limit of Gunn's solution. The asymptotic form remains reasonably accurate almost as far upstream as one semispan from the leading edge. Walter G. Vincenti, USA

1239. Rand, Robert C., Prandtl-Meyer flow behind a curved shock wave, *J. Math. Phys.* **29,** 2, 124–132, July 1950.

The flow downstream of a curved shock in a plane uniform stream cannot be irrotational. Author constructs a nonirrotational free stream, and a curved shock in it, in such a way that the downstream flow becomes an irrotational Prandtl-Meyer flow. Then he compares this free stream with the uniform stream. The construction is based upon the known shock equations for ideal gases. R. Sauer, Germany

1240. Kantrowitz, Arthur, and Neice, Stanford E., Stabilization of shock waves in channel flow, *J. aero. Sci.* **17,** 5, 316–317, May 1950.

Stabilization of the normal shock in supersonic diffusers is investigated by means of surge chambers communicating with the channel through slits just ahead of stable position of the shock. When a pressure pulse from rear moves the shock forward, the chambers absorb mass from behind shock, stopping it before the unstable position is reached. Result of a calculation is given, assuming an infinite reservoir, showing that slits 40% of area of second throat are required to double the ability of channel to absorb compression pulses. Experiments are performed in a small supersonic diffuser at Mach number 1.65 which verifies the theoretical expectations. Walker Bleakney, USA

1241. Lorell, Jack, An observation concerning the Rankine-Hugoniot relations, *J. aero. Sci.* **17,** 5, 317, May 1950.

A short proof is presented of the fact that the Rankine-Hugoniot relations imply increasing entropy across a normal shock wave, going downstream. The minimum Mach number after a normal shock is also derived. L. L. Cronvich, USA

1242. Bernard, Jean J., On the continuous and one-dimensional flow through a straight or oblique shock wave (in French), *C. R. Acad. Sci. Paris* **230,** 14, 1339–1340, Apr. 1950.

Note deals with an extension of work of Roy in France, of Morduchow and Libby in United States, to the calculation of normal and oblique shock-wave characteristics. An arbitrary value of the Prandtl number is considered and viscosity is assumed to be an exponential function of temperature. Solutions for the downstream velocity and temperature are obtained by approximate methods. M. J. Thompson, USA

1243. Gilmore, F. R., Plesset, M. S., and Crossley, H. E., Jr., The analogy between hydraulic jumps in liquids and shock waves in gases, *J. appl. Phys.* **21,** 3, 243–249, Mar. 1950.

Water-table observations of regular and Mach reflections were made by authors using essentially the same experimental setup as was used by Einstein and Baird in earlier investigations. Jump strengths were measured by resistance depth gages and flow patterns were recorded on refraction photograms. Wave generators were reservoirs into which a volume of water could be raised above the free surface and then discharged quickly through an adjustable horizontal slot. It is suggested that observed discrepancies between measured and theoretical propagation velocities of strong hydraulic jumps may be attributed to this technique of wave generation, which causes released water to "override" surface of the still water. Weak jumps were found to have an undulatory profile; stronger jumps, a smoother profile; and very strong jumps showed irregular roughness. When surface-tension reducing agents were added to water, wave fronts became steeper and strong jumps smoother, but amplitude of undulations in weaker jumps was increased. Agreement between theory and experiments was found to be fair for regular intersections but poor for Mach intersections. Such discrepancies have also been

observed in air [AMR **3**, Rev. 1530] but they are considerably greater in the hydraulic case. It is noted that thickness of the jumps is not negligible compared to their radius of curvature as in compression shocks and as assumed in simple theory. Such sources of error, which are peculiar to hydraulic jumps, limit the utility of the water table as an analog device.

Joseph V. Foa, USA

1244. **Miles, Edward R. C., On a family of oblique shock curves,** *J. aero. Sci.* **17,** 3, 157–158, Mar. 1950.

A simple parametric representation is derived for curves of $M \sin\sigma$ = constant (where σ is the angle of oblique shock). If these curves are plotted in polar coordinates together with the shock polars, resulting diagram affords a means of graphically evaluating pressure, temperature, and density ratios across the shock, in addition to the kinematic relations given by the shock polar.

John V. Becker, USA

1245. **Whitham, G. B., The behaviour of supersonic flow past a body of revolution, far from the axis,** *Proc. roy, Soc. Lond. Ser. A, Math. & Phys. Sci.* **201,** 1064, 89–109, Mar. 1950.

For a general body of revolution of finite length, asymptotic shapes and strengths of the front and rear shocks are described, using expansions of the physical quantities for large distance from the axis. Between shocks, pressure falls approximately linearly, the slope being independent of particular body considered. In the special case of slender pointed bodies, theory is used to modify the linearized approximation. Theoretical results are compared with results of experimental observations.

R. Sauer, Germany

1246. **Pai, S. I., On the vorticity of the flow behind curved shock,** *J. aero. Sci.* **17,** 3, 188–189, Mar. 1950.

"One of the assumptions usually made in calculation of flow involving shock wave is that the vorticity introduced into the flow by the shock wave is extremely small and negligible. [This paper shows] that this assumption, in general, is not accurate and that, in most practical cases, the vorticity introduced by the shock should be considered."

Author's proof consists of showing that the perturbation velocities immediately behind a shock wave and the vorticities are of same order of magnitude. A. Vazsonyi, USA

1247. **Sauer, R., Propagation laws of weak shocks in gases** (in German), *Ingen.-Arch.* **18,** 4, 239–241, 1950.

Simple derivation (by K. Oswatitsch) of asymptotic laws of propagation of plane shock waves is extended to cylindrical and spherical case assuming (a) medium: ideal gas with constant isobaric and isochoric heat capacities, (b) rate of change of entropy jump across shock front is constant, (c) for the time t = constant, pressure is proportional to density behind shock front. Given an initial pressure-distance curve, asymptotic expressions are derived for shock line and pressure-distance curves at $t > 0$. Paper transcends acoustic approximation. Author notes agreement with experiment for t large enough. Theory useful for qualitative studies only.

Ray C. Makino, USA, and L. N. Enequist, USA

1248. **Whitham, G. B., The propagation of spherical blast,** *Proc. roy. Soc. Lond. Ser. A,* **203,** 1075, 571–581, Oct. 1950.

The acoustic solution for outgoing spherical waves is of the form $f(z)r^{-1}$ where r is distance from the origin at time t, $z(=a_0t - r)$ equals constant defines the characteristic, and a_0 is the sonic speed. Author seeks solution to the nonlinear isentropic equations valid for large r in series of form $\Sigma A_n r^{-n} + B_n \ln r\, r^{-n}$ with coefficients constant on each *exact* characteristic, and obtains equation of exact characteristic in the process. Shocks are considered by patching isentropic solution with Rankine-Hugoniot relations. The connection with linearized theory is shown for waves of small amplitude. Author states that linearized theory becomes invalid at large r because approximate characteristic is used instead of exact characteristic. Hence, a solution uniformly valid for all r is obtained from linear solution by substituting equation of exact characteristic obtained in previous analysis wherever approximate expression appears.

Theodore R. Goodman, USA

1249. **Miles, John W., On the compressibility correction for subsonic unsteady flow,** *J. aero. Sci.* **17,** 3, 181–182, Mar. 1950.

A transformation is found that reduces the linearized boundary-value problem of an oscillating finite wing in compressible flow to a related one in incompressible flow, provided the oscillation is slow enough. Consequently, a formula can be written for the pressure distribution on the original wing; it involves the related nonstationary case and two steady-flow distributions. The analogous problem for infinite wings was treated earlier by the same author [*J. aero. Sci.* **16,** 7, 440, July 1949; AMR **4**, Rev. 284] and a large logarithmic frequency term was found. Author states here that his new results show this term to be important only for very large aspect ratios. W. R. Sears, USA

1250. **Kiebel, I. A., Some studies on the flow of a gas in the region of transition through the velocity of sound,** *Nat. adv. Comm. Aero. tech. Memo.* 1252, 12 pp., Apr. 1950.

The basic equation for the transonic region as used by Christianovich, Frankl, and Falkovich is briefly discussed.

H. Yoshihara, USA

1251. **Sears, W. R., Transonic potential flow of a compressible fluid,** *J. appl. Phys.* **21,** 8, 771–778, Aug. 1950.

Author reviews present status of solutions of transonic-flow problems. Each solution so far published is critically examined for its physical significance and weakness. Significance and probable correctness of stability considerations is examined, and the importance and direction of further theoretical and experimental studies are discussed. H. W. Emmons USA

1252. **Tollmien, W., Theory of characteristics,** *Nat. adv. Comm. Aero. tech. Memo.* 1242, 28 pp., 1949.

[Translated from "Technische Hochschule Dresden, Archiv" 44/2, chap. 2; date of original publication not given.] An expository article on the characteristics theory for second-order quasi-linear partial differential equations, geared for practical applications and including an extension of the Prandtl-Busemann method for the equation of gas dynamics [Stodola Festschritt, Zurich, pp. 499–509, 1929] to the general case. L. Bers, USA

1253. **Ursell, F., and Ward, G. N., On some general theorems in the linearized theory of compressible flow,** *Quart. J. Mech. appl. Math.* **3,** part 3, 326–348, Sept. 1950.

Linearized equations of motion for steady isentropic flow past thin bodies with sharp trailing edge are used to evaluate the leading terms of the force acting on the body where there is a vortex wake. With certain additional assumptions the linearized theory is adequate for calculation of drag and lift, but not for the lateral force, except in special cases. A quadratic identity shows that there is at most one solution with given boundary conditions. The corresponding bilinear identity is used to obtain relations between the flow and the reversed flow past the same body.

These relations hold for both subsonic and supersonic flow and include all flow-reversal theorems obtained by M. M. Munk and other earlier writers.

From authors' summary by R. Sauer, Germany

1254. Kolodner, I., On the linearized theory of supersonic flows through axially symmetrical ducts, *Comm. pure appl. Math.* **3,** 2, 133–152, June 1950.

It is proved that a unique solution of linearized supersonic-flow problem through axially symmetrical ducts exists if the first, second, and third derivatives of the duct function (= diameter of the slowly varying cross section) satisfy certain conditions of continuity. Author gives the solution explicitly, using Laplace transforms, and investigates what happens if some of the continuity conditions are not fulfilled, e.g., if the tangent and the curvature of the duct curve have jump discontinuities. Previous research of H. L. Ludloff and F. Reiche [*J. aero. Sci.* **16,** p. 5, 1949; AMR **1,** Rev. 315], Ch. Mack [Doct. thesis 1947] and G. N. Ward [*A.R.C.* R. & M. 2183] are discussed, and the analogous problem of two-dimensional symmetrical ducts is treated shortly.

R. Sauer, Germany

1255. Jack, John R., Theoretical wave drags and pressure distributions for axially symmetric open-nosed bodies, *Nat. adv. Comm. Aero. tech. Note* 2115, 35 pp., June 1950.

Results are computed by two methods, the linearized method of characteristics and the method of supersonic sources, the second being the more rapid. Both conical and curved cowlings are considered, and case in which the shock from a central body passes upstream of cowling lip is included. Graphs show variation of pressure and wave drag coefficients with free-stream Mach number, fineness ratio, area ratio, and position of central shock. A formula is proposed to fit the pressure variation on conical cowlings. Maurice Holt, England

1256. Moore, Franklin, Linearized supersonic, axially symmetric flow about open-nosed bodies obtained by use of stream function, *Nat. adv. Comm. Aero. tech. Note* 2116, 30 pp., June 1950.

Author shows that the linearized flow past an open-nosed body, whose surface is not generally near the axis of symmetry, is better treated with Stokes' stream function than with the velocity potential. Behavior of the stream function near the body lip is investigated in detail and found to be well represented by an axial distribution of half-power supersonic sources. Using this, pressure near the lip and strength and inclination of the shock wave starting from it are determined and found to be correct to the first order. Surface pressure further downstream is not so accurately given by this distribution. Maurice Holt, England

1257. Miles, John W., Some relations between harmonic and transient loadings of airfoil, *J. aero. Sci.* **17,** 10, 671–672, Oct. 1950.

By considering the duality between the responses of a linear system to harmonic and step function excitations, some theorems regarding the transient loading of thin airfoils are obtained. Results of their application to several problems are presented briefly. Keith C. Harder, USA

1258. Vavra, M. H., Steady flow of nonviscous elastic fluids in axially symmetric channels, *J. aero. Sci.* **17,** 3, 149–156, 172, Mar. 1950.

Author's theory is applicable to flow with constant total energy between two arbitrary coaxial surfaces of revolution. Velocity at any point is resolved into a component in the meridian plane (meridional velocity) and a component normal to this plane (tangential velocity). Using a transformation of the Eulerian equation of motion, an expression is obtained allowing meridional velocity to be determined for an arbitrary distribution of tangential velocity. From this expression an iteration method is developed for calculating the flow with given entry conditions and boundaries. Method can be used for flow with vorticity. As an example, flow in a compressor inlet duct is calculated for flow with and without vorticity. Results show important differences between these two cases. With vorticity, for given boundaries and tangential velocity distribution, there is a lower limit of meridional velocity (i.e., of mass flow) below which stable flow following the boundaries is not possible. W. A. Mair, England

1259. Poritsky, H., Sells, B. E., Danforth, C. E., Graphical, mechanical, and electrical aids for compressible fluid flow, *J. appl. Mech.* **17,** 1, 37–46, Mar. 1950.

Methods are described for studying two-dimensional and axially symmetrical isentropic flow of nonviscous compressible fluids. In the graphical method, the equipotential and streamlines are plotted, using the known theoretical relations between the sides of the rectangles formed by these lines. The work can be facilitated by use of a network of wires and pins, with a simple device for checking the length-width ratios of the rectangles. In the method based on the electrical analogy, problem is set up as a resistance network on a d.c. board. With any of these methods successive approximations are necessary to obtain the final solution. Methods are suitable for solving problems where no analytical solutions are known and the boundary conditions are well defined, e.g., cascades of blades. Several illustrative examples are given. In cascade problems local velocities can be determined within about 3%. W. A. Mair, England

1260. Illingworth, C. R., Unsteady laminar flow of gas near an infinite flat plate, *Proc. Camb. phil. Soc.* **46,** part 4, 603–613, Oct. 1950.

Solutions of the compressible problem of unsteady laminar flow near an effectively infinite plate set into motion in its own plane are obtained by subjecting the boundary-layer equations to von Mises' transformation. Gravity produces a secondary flow, which, however, may be negligible in most cases. Solutions are obtained for flows with arbitrary Prandtl number σ, constant plate temperature and a velocity which is either uniform, or, with dissipation neglected, nonuniform. Explicit solutions are given for the case $\mu_r \propto T_r$. Reviewer remembers that the corresponding incompressible problem, solved and tabulated by H. Görtler [*Ingen.-Arch.* **14,** p. 300, 1944] leads to same results. Further solutions are gained for associated problem of diffusion from a vortex sheet. For nonuniform motion of a heat-insulated plate, dissipation not being negligible, a solution is obtained when σ is 1 and $\mu_r \propto T_r$. Gravity becomes relatively important for the free convection current and a solution is given for $\mu_r \propto T_r$ and neglected dissipation, when the free convection current is set up at a stationary plate by a constant temperature difference between the plate and surrounding gas. Walter Wuest, Germany

1261. Tifford, Arthur N., On the surface effects of a compressible laminar boundary layer, *J. aero. Sci.* **17,** 3, 187–188, Mar. 1950.

Referring to the paper by D. R. Chapman and M. W. Rubesin [see AMR **3,** Rev. 2732] author shows that the Blasius profile as well as the laws of friction and heat transfer persist effectively even at Mach numbers as high as 5 if fluid properties are taken as those at the plate's surface. A brief review of relevant literature should confirm this statement. Albert Betz, Germany

1262. **Behrbohm, Hermann, An approximate theory of the unsymmetrical transonic flow through a Laval nozzle** (in German), *Z. angew. Math. Mech.* **30**, 4, 101–112, Apr. 1950.

The two-dimensional Tricomi problem is shown to reduce to the (parabolic) problem of heat conduction for three types of flow, viz., simple source flow, simple vortex flow, and a family of nozzle flows including one (practicable) case with symmetry about a straight streamline. For the last one, a solution is given in form of Fourier series with respect to the stream function, and the coefficients are determined from the velocity distribution upstream. For a special such distribution the solution reduces to the known first two terms in the double Taylor series in the flow plane of the velocity near the sonic point of the axis.

R. E. Meyer, England

1263. **Schrenk, O., Diabatic gas flow with constant cross section** (in German), *Ingen.-Arch.* **18**, 4, 272–276, 1950.

1264. **Gray, Vernon H., Improvements in heat transfer anticing of gas-heated airfoils with internal fins and partitions,** *Nat. adv. Comm. Aero. tech. Note* 2126, 44 pp., July 1950.

Experimental investigation at airspeeds up to 300 mph in wet and dry airstreams was made to find heat-transfer rates of three gas-heated hollow metal airfoils, each having a different arrangement of metal internal fins. Although width of wind tunnel was less than 2 in., results are believed to be reliable. One arrangement of tapered metal fins attached to inner forward portion of heating-air duct, combined with a reduction in area of the duct, increased the surface heating rate at the stagnation point by a factor of 3.5, when compared with the duct with no internal fins or reduction in area; increase was greatest at the stagnation point. Tapered internal fins with a reduced gas-flow passage resulted in higher heat-transfer rates than either no fins or constant thickness fins with no reduction in duct width. Neal Tetervin, USA

1265. **Shaw, B. W. Bolton, Nose controls on delta wings at supersonic speeds,** *Coll. Aero. Cranfield Rep.* **36**, 56 pp., May 1950.

Linearized theory is used to evaluate the effectiveness of flat triangular control surfaces hinged along the leading edges of flat delta wings, with the hinge lines meeting at the apex. For low aspect ratio wings (say $A \leqslant 1$) these controls are too ineffective to be of interest in aircraft applications. The effectiveness, of course, increases with increasing aspect ratio but is always less than that theoretically obtainable with trailing edge controls of same relative area. At an aspect ratio of 4.0, for example, the effectiveness is about half that of the trailing edge type.

John V. Becker, USA

1266. **Hughes, G. F., Measurements of a twisting moment due to aileron deflection on model Mustang wing at high speed,** *Aero. Res. Coun. Rep. & Mem.*, R. & M. no. 2359, 10 pp., 1950.

Results of paper are based on experimental values of half-wing lift and pitching moment given as functions of aileron angle deflection, and at free-stream Mach numbers equal to 0.4, 0.74, and 0.81. Max A. Heaslet, USA

1267. **Stewart, H. J., and Li, Ting-Yi, Periodic motions of a rectangular wing moving at supersonic speed,** *J. aero. Sci.* **17**, 9, 529–539, Sept. 1950.

In the past, calculations for determination of the air forces and moments on an oscillating airfoil in a main stream of uniform supersonic speed have been performed mostly on the basis of an analysis of a two-dimensional wing of infinite span. A satisfactory theory of the corresponding problem for a three-dimensional wing with finite span is not yet available. Authors developed some general integral expressions for the disturbance-velocity potential of a three-dimensional oscillating wing with both purely and mixed supersonic boundary conditions. Method used was essentially that of Evvard. These general results are applied here to determine the lift and moment (due to lift) acting upon a thin oscillating wing of rectangular plan form moving at supersonic speed. Detailed analysis in case of vertical oscillations is presented. Progress in the analysis of pitching oscillations is also indicated. From authors' summary by B. Etkin, Canada

1268. **Ribner, Herbert S., Some conical and quasi-conical flows in linearized supersonic-wing theory,** *Nat. adv. Comm. Aero. tech. Note* 2147, 61 pp., Aug. 1950.

Paper treats a number of conical and quasiconical linearized supersonic flows which are useful in calculating change of lift-distribution under the influence of the subsonic trailing edge of the wing.

Two methods of source integration are used: (1) Evvard's membrane analogy, and (2) Schlichting's superposition technique. Both involve Abel's integral equation in the direct problem. In addition, both involve interaction in the flow field between subsonic trailing edge and tip Mach cone. It is too complicated to derive simple analytic solutions which can be easily applied to the damping problem in roll and pitch. Present approach, along the line of Evvard, is, first, to find the integral equations of some simplified flow field neglecting interaction. Next, along the line of Schlichting's superposition, exact flow pattern is used to formulate the integral equation. In all the cases considered, the singular kernels are too complicated to obtain solutions easily. Some theorems are developed by Moore so that differentiation of the singular improper integral is permissible under suitable restrictions. After such differentiation, the integral equation becomes Prandtl's equation in lifting-line theory. Both methods agree well for large values of the ratio of swept angle of trailing edge to tangent of Mach angle.

Finally, the equivalence of Evvard's source integration method and Goodman-Marels' method of doublet distribution is shown.

C. C. Chang, USA

1269. **Ribner, Herbert S., On the effect of subsonic trailing edges on damping in roll and pitch of thin sweptback wings in a supersonic stream,** *Nat. adv. Comm. Aero. tech. Note* 2146, 27 pp., Aug. 1950.

The usual sweptback plan form may be regarded as derived from a delta wing by clipping off streamwise tips and cutting out the trailing edge. For subsonic leading edges, effect upon damping in pitch and roll of clipping off tips was evaluated by means of linearized theory in an earlier paper [AMR **2**, Rev. 1424]. The additional effect of cutting out a subsonic trailing edge is now calculated, using certain conical and quasiconical flows published elsewhere (see preceding review). Results are limited to plan forms and Mach numbers for which the trailing-edge disturbance does not cross the leading edge. Only the major portion of lift in the wake is canceled, and secondary interference between tip and trailing edge is neglected. Validity of these approximations is discussed. Extension of the method to calculation of rolling moments due to sideslip and yaw is indicated.

M. D. Van Dyke, USA

1270. **Oswatitsch, Klaus, Progress in gas dynamics** (in German), *Acta Phys. Austriaca* **3**, 1–21, 1949.

A short introductory presentation of the field of gas dynamics, dealing with the essential concepts rather than with detailed solution methods. Discussion starts with the concept of Mach num-

ber, continues through the nature of stationary subsonic and supersonic flows, touches the transonic field, and goes into shock waves, nonsteady flow, and flows with viscosity, turbulence, and heat conduction. Particular attention is paid to the zone of influence concept and to a discussion of the realms of application of gas dynamics. W. D. Hayes, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 1209, 1222, 1234, 1261)

1271. Yaglom, A. M., Homogeneous and isotropic turbulence in a viscous compressible fluid (in Russian), *Izvestiya Ser. Geogr. Geofiz.* **12**, 501–522, 1948.

Author considers decay of isotropic turbulence in a compressible fluid neglecting the third-order correlation functions. Equations for the second-order correlations are then developed and reduced to a system which determines seven such functions. One of these corresponds to that for an incompressible field, the other six to an irrotational field. Considerable discussion is given to the fact that the latter field contains acoustic waves propagating with acoustic velocity and a nonpropagating entropy wave. The Fourier transforms of the correlation functions are written down and the decay rate is determined as a function of frequency. [Since the transfer terms were omitted, all of the results could have been based as readily on a discussion of a random acoustic field in a viscous field as on a turbulent field.]

G. F. Carrier, USA

1272. Chandrasekhar, S., The decay of axisymmetric turbulence, *Proc. roy. Soc. Lond. Ser. A*, **203**, 1074, 358–364, Oct. 1950.

The decay of axisymmetric turbulence is investigated under conditions appropriate to the final period of turbulent decay, i.e., inertial terms in equations of motion negligible. On this assumption, explicit forms are obtained for the two scalar functions determining the correlation tensor in terms of initial values of these functions, and it is confirmed that correlation tensor so defined asymptotes to form given by Batchelor. Finally, an interesting special case of axisymmetric turbulence is considered, consisting of superposition of two noninteracting fields of turbulence, one an isotropic field satisfying the von Kármán-Howarth equation, and the other an axisymmetric field. It would be interesting to know whether this special case represents any real turbulent field, for it is a type of axisymmetric turbulent motion possible in principle at finite Reynolds numbers.

A. A. Townsend, England

1273. Pearcey, H. H., The indication of boundary-layer transition on aerofoils in the N.P.L. 20 in. × 8 in. high speed wind tunnel, *Aero. Res. Coun. Curr. Papers*, C. P. 10, 8 pp., Dec. 1948, published 1950.

Paper presents description of "liquid film" evaporation method for determining transition in a specific wind tunnel, and a qualitative explanation of shadowgraph pictures of boundary layers. It is shown that liquid-film observations and surface-pressure measurements can be made simultaneously and that shadow photographs may indicate the approximate location of transition.

W. F. Davis, USA

1274. Ferrari, Carlo, Study of the boundary layer at supersonic speeds in turbulent flow: Case of flow along a flat plate, *Quart. appl. Math.* **8**, 1, 33–57, Apr. 1950.

Paper attempts to apply methods used in incompressible turbulent boundary-layer theory to the corresponding supersonic problem. Taking many assumptions into account, it gives theoretical formulas for distribution of velocity within boundary layer, drag depending on Reynolds number and variation of thickness of boundary layer. Equation of continuity contains an empirical parameter for which a plausible value is taken from Frössel's experiments on sub- and supersonic streams in pipes. This parameter depends on Mach number, being zero for Mach number zero. Resulting laws for drag and boundary-layer thickness are formally identical with corresponding incompressible ones. Numerical results are given for $M = 1.5$ and 2.

F. W. Riegels, Germany

1275. Schlichting, H., Turbulence and heat stratification, *Nat. adv. Comm. Aero. tech. Note* **1262**, 55 pp., Oct. 1950.

Translation from *Z. angew. Math. Mech.* **15**, 6, Dec. 1935.

1276. Rotta, J., Velocity law of turbulent flow valid in the neighborhood of a wall (in German), *Ingen.-Arch.* **18**, 4, 277–279, 1950.

With his mixing length assumption Prandtl gave the following expression for the shearing stresses τ near a wall: $\tau = \rho\,[l^2\,|dU/dy| + \nu]\,dU/dy$, where U is average velocity at distance y from wall, ρ density, ν kinematic viscosity, l mixing length. Assuming a linear relation between l and y and putting τ constant and equal to shearing stress at the wall, author integrates this equation and obtains an approximate solution for the velocity distribution. For smooth walls one has to distinguish between two ranges, one as a laminar sublayer (thickness δ) without turbulence ($l = 0$) and another turbulent layer for which author assumes $l = \kappa(y - \delta)$ with κ as a universal constant. The velocity profile obtained by integration fits very well with the measurements of H. Reichardt. For rough walls the boundary conditions are not so simple. Author defines a plane of reference for the distance y from the wall, where the velocity U disappears on the average. Small roughness creates an undulation of the laminar layer which means a decrease of its average thickness. This could be proved by the measurements on walls of different roughnesses made by T. Nikuradse. For medium roughness a laminar layer does not exist any more, but the turbulence even in the reference plane has a finite value. Finally for extreme roughness its intensity at $y = 0$ becomes so large that the viscosity has no influence at all. This can be realized by assuming a length l_0 depending on the roughness and introducing into the shearing stress equation, the expression: $l = l_0 + \kappa y$. Author gives the numerical solution for different roughnesses, but the available experimental material is not sufficient for a comparison with the theory.

K. Pohlhausen, USA

1277. Ross, Donald, and Robertson, J. M., Shear stress in a turbulent boundary layer, *J. appl. Phys.* **21**, 6, 557–561, June 1950.

Fediaevsky's shear stress theory [*Central Aero-Hydrodynamical Institute, Moscow*, ref. rep. **282**, 1936; also *NACA T.M.* 822, 1937] is modified by changing one of the boundary conditions and the form of the polynomial for the shear stress distribution to include the space history of the turbulence. Calculations for one case show somewhat better agreement with experiment than Fediaevsky's theory gives. J. M. Wild, USA

1278. Meksyn, D., Stability of viscous flow over concave cylindrical surfaces, *Proc. roy. Soc. Lond. Ser. A*, **203**, 1073, 253–265, Sept. 1950.

Author investigates stability of two-dimensional boundary-layer flow on a concave surface when it suffers small three-dimensional disturbances in the form of vortexes with their axes in the direction of flow (Görtler's problem). A sixth-order dif-

ferential equation is set up for the variation of the disturbance amplitude with distance (y) from the surface. Following author's method for rotating cylinders [title source, **187**, 115–128, 1946] asymptotic solutions valid for large values of $\mu(=R_\delta^2\delta/r)$ and $\sigma(=2\pi\delta/\lambda)$ where δ = boundary-layer thickness, R_δ = Reynolds number, r = radius of curvature of boundary, λ = disturbance wave length, are sought in the form $C(y/\delta)\exp[\xi(y/\delta)]$, where $C(y/\delta)$ varies only slowly and $\xi(y/\delta)$ contains the large parameters μ and σ. C and ξ are found by retaining only terms of the two highest orders in the differential equation.

Approximate calculations are carried out for velocity profile $u=y/\delta, 0\leqslant y\leqslant\delta; u=1, y>\delta$. Surface boundary-layer conditions and requirements of smooth behavior at the boundary-layer edge provide a sixth-order determinantal condition relating μ and σ for neutral disturbances. Critical minimum values of parameters for the two lowest modes are $\alpha\theta=0.400$, $R_\theta(\theta/r)^{1/2}=3.65$, and $\alpha\theta=0.75$, $R_\theta(\theta/r)^{1/2}=10.7$, where θ = momentum thickness (cf. Görtlers approximate integral equation method which gave $R_\theta(\theta/r)^{1/2}=0.58$, $\alpha\theta=0.14$). Rate of amplification in neighborhood of first neutral stability curve is also estimated. A. F. Pillow, Australia

Aerodynamics of Flight; Wind Forces

(See also Revs. 1032, 1126, 1238, 1249, 1264, 1266, 1268, 1269, 1311, 1313, 1325, 1333)

1279. **Weil, Joseph, and Sleeman, William C., Jr., Prediction of the effects of propeller operation on the static longitudinal stability of single-engine tractor monoplanes with flaps retracted,** *Nat. adv. Comm. Aero. Rep.* 941, 20 pp., 1949.

Paper presents conveniently arranged survey of method for computing effects of power on wing fuselage and tail contributions to stability. Method can be used when power-off lift and pitching moment data (preferably obtained from wind-tunnel tests) are known and based on wind-tunnel investigation of 28 powered models.

Special attention is paid to calculation of change in downwash due to power; an empirical method based on experimental results is adopted. Calculated longitudinal characteristics are in very good agreement with experiment. It is stated, however, that all models considered represent highly powered fighter-type airplanes. Limits of geometric variables between which computing method is considered to be reliable are given, although possibility exists that those limits are too narrow.

J. Buhrman, Holland

1280. **Miles, John W., On the equations of longitudinal stability,** *J. aero. Sci.* **17,** 12, 815–816, Dec. 1950.

Author, by analyzing the stability quartic in terms of dimensionless parameters, shows that for the case of power-off flight at small angles of attack α, the only unsteady-flow term of importance is $m_{\dot\alpha}\dot\alpha$ where $\dot\alpha$ denotes a vertical acceleration.

A. Petroff, USA

1281. **Frick, C. W., and Chubb, R. S., The longitudinal stability of elastic swept wings at supersonic speed,** *J. aero. Sci.* **17,** 11, 691–704, Nov. 1950.

Application of linearized lifting surface theory to elastic tapered swept wings of high aspect ratio, with leading edges swept behind Mach lines, and trailing edge sonic or supersonic. Accelerated flight conditions only are considered.

Straight flexural axis is assumed. Parabolic wing deflection curve, twist of streamwise section varies linearly across span. Pressure distribution is found by superposition of conical flow for (1) rigid wing at angle of attack of root section, (2) twisted wing at zero angle of attack.

Ratio of lift, pitching moment, span load distribution of elastic wing to that of rigid wing are given in terms of wing geometry, maximum design stress, modulus of elasticity, shearing modulus, and maximum design load factor. Above results are compared with strip theory method [*NACA T.N.* 1566; AMR **3,** Rev. 1135]. This form of strip theory gives good accuracy.

To investigate assumptions of parabolic deflection curve, strip theory method is used with matrix methods of Diederich [*NACA T.N.* 1876; AMR **3,** Rev. 561]. Assumption of parabolic deflection gives good estimate of longitudinal stability characteristics.

Curves are given for shift of neutral point and for change in lift curve slope in terms of dynamic pressure for given sweptback wing at $M=1.414$ for (1) pure bending, (2) bending and torsion. Results are compared with incompressible flow calculations [*NACA T.N.* 1772]. At $M=1.414$, there is slightly greater loss in lift curve slope and larger shift in neutral point. However, for wings swept within Mach cone these effects are primarily functions of flight dynamic pressure. A. W. Babister, England

1282. **Sternfield, Leonard, Some effects of nonlinear variation in the directional-stability and damping-in-yawing derivatives on the lateral stability of an airplane,** *Nat. adv. Comm. Aero. tech. Note* 2233, 19 pp., Nov. 1950.

Values of the two derivatives are assumed to be zero for small angles of sideslip and to vary linearly outside this dead zone. Using the Laplace transform, separate solutions are obtained for motion inside and outside of dead zone; from these, variations of sideslip angle following an initial disturbance were computed by a stepwise procedure. It is found that under certain conditions the rates of damping are different for large and small amplitudes of motion, with very little damping at small amplitudes. Period of the oscillation increases with time. Dana Young, USA

1283. **Murray, Harry E., and Grant, Frederick C., Method of calculating the lateral motions of aircraft based on the Laplace transform,** *Nat. adv. Comm. Aero. tech. Note* 2129, 54 pp., July 1950.

The lateral motions of aircraft are obtained by means of the Laplace transform which gives solutions expressed in terms of elementary functions for the free and forced motions.

From authors' summary

1284. **Schade, Robert O., Free-flight-tunnel investigation of dynamic longitudinal stability as influenced by the static stability measured in wind-tunnel force tests under conditions of constant thrust and constant power,** *Nat. adv. Comm. Aero. tech. Note* 2075, 14 pp., Apr. 1950.

Results are presented that tend to confirm previous studies which have indicated that the longitudinal steadiness of airplanes is affected to a much greater extent by changes in constant-thrust static margin than by changes in constant-power static margin. Harvard Lomax, USA

1285. **Martin, Norman J., and Holzhauser, Curt A., Analysis of factors influencing the stability characteristics of symmetrical twin-intake air-induction systems,** *Nat. adv. Comm. Aero. tech. Note* 2049, 16 pp., Mar. 1950.

Methods are presented for predicting the ratios of inlet to free-stream velocity for which flow instability and reversal occur. Instability and reversal are found to depend on characteristics of static-pressure recovery at junction of the two inlet ducts, but cannot be determined from total- or ram-pressure recovery characteristics. Theory agrees well with limited experimental results available at present. N. H. Johannesen, England

1286. Huffschmid, A., Investigation of the model ME 210 in the spin wind tunnel of the DVL. Fourth partial report—model with long fuselage and with a vee tail, *Nat. adv. Comm. Aero. tech. Memo.* 1222, 27 pp., Apr. 1950.

Contrary to expectation, configuration tested showed irregular steady spins; the rudder effectiveness with regard to spin recovery was inferior to standard tail surfaces. Configuration tested was the most unfavorable one of four fuselage-tail combinations tested in this series. Reviewer questions the value of the report for following reasons: (a) Three essential references, describing tests of the other combinations, are apparently not available; (b) no scaled three-view of model tested is given; (c) slat sizes and locations are not given; (d) test facility and techniques used are neither described nor referenced; and (e) notation is not defined. H. P. Liepman, USA

1287. Beckhardt, Arnold R., A theoretical investigation of the effect on the lateral oscillations of an airplane of an automatic control sensitive to yawing accelerations, *Nat. adv. Comm. Aero. tech. Note* 2006, 27 pp., Jan. 1950.

Investigation described in title included calculations of effect of time lag in a control of this type on damping of lateral oscillations of a typical high-speed airplane, and also calculations of the effect on damping of varying the ratio of rudder deflection to yawing acceleration.

Results indicate that a control of this type can successfully damp the lateral oscillations through a reasonable range of time lag. From author's summary by Andrew Vazsonyi, USA

1288. Roberts, Howard E., and Langtry, B. D., The influence of design parameters on the performance of subsonic air inlets, *J. aero. Sci.* **17,** 7, 429–435, 456, July 1950.

The influence, as determined by experiment, of aspect ratio, degree of protuberance, ramp profile, boundary-layer thickness, and bleed position upon the inlet-total-pressure recovery, critical Mach number and boundary-layer-bleed-total-pressure recovery for an inlet with boundary-layer bleed is presented in the form of graphs and design charts. Keith C. Harder, USA

1289. Amer, Kenneth B., Theory of helicopter damping in pitch or roll and a comparison with flight measurements, *Nat. adv. Comm. Aero. tech. Note* 2136, 25 pp., Oct. 1950.

Damping moments about the helicopter c.g. produced during pitching or rolling by a rotor with flapping hinges on the rotor shaft is shown theoretically to vary directly with hub height, inversely with rotor speed and blade mass factor, and linearly with the parameter $\theta/(C_T/\sigma)$ (θ is collective pitch, C_T thrust coefficient, and σ solidity ratio). Main result is that these moments become destabilizing when $\theta/(C_T/\sigma)$ exceeds about 3. Consequently, present-day helicopters with conventional control systems tend to have low damping at high speed, and may even have negative damping during a maneuver when the normal acceleration falls well below 1g. Author suggests use of offset flapping hinges on high-speed high-powered helicopters as one way of avoiding such instability. Results of a flight test are shown to agree with theoretical predictions. Both theory and test indicate that calculations based on assuming the rotor-force-vector tilt to be equal to the tip-path-plane tilt during rolling or pitching may give highly misleading results.

Morris Morduchow, USA

1290. Stepniewski, W. Z., Introduction in helicopter aerodynamics, Morton, Pa., Rotorcraft Publishing Committee, 1950, 158 pp. $2.50.

Booklet fulfills author's intention of presenting, in a simplified manner, the fundamental aerodynamic phenomena involved in the operation of rotary wing aircraft. Prior knowledge of rotary wing craft is not required although it is assumed that reader is familiar with the basic theories of flight. While no new material is introduced, presently accepted theories are compiled in a clear, concise manner.

Chapter I briefly reviews the conventional fundamental aerodynamic principles. Next three chapters are devoted to simple momentum, blade element, and vortex theories as applied to propellers. Chapter V deals with practical methods of calculating rotor thrust in hovering and vertical flight. Final three chapters are concerned with performance in hovering flight, vertical ascent, powered forward flight, and autorotation. References to the literature are given throughout.

Reviewer feels that this is a worthy contribution to the rather meager number of texts devoted to the fundamentals of this interesting type of craft. Leonard Goland, USA

1291. Summers, James L., and Treon, Stuart L., The effects of amount and type of camber on the variation with Mach number of the aerodynamic characteristics of a 10-percent-thick NACA 64A-series airfoil section, *Nat. adv. Comm. Aero. tech. Note* 2096, 66 pp., May 1950.

Results are presented of wind-tunnel tests of airfoil sections cambered for design section lift coefficients of 0.3, 0.6, and 0.9 on the NACA $a = 1.0$ mean line, and for 0.3 and 0.6 on the NACA $a = 0.4$ mean line. Mach numbers were varied from 0.3 to approximately 0.9, with corresponding Reynolds numbers from 1.0×10^6 to 2.0×10^6.

It was found that increases in camber resulted in large increases in lift- and drag-divergence Mach numbers at high lift coefficients and in augmentation of the maximum lift coefficient. The variation of lift-curve slope with Mach number was most favorable, at a given lift coefficient, for the airfoil having a design section lift coefficient equal to the given value. At low and moderate Mach numbers, the improvements in lift-drag ratio ordinarily expected of camber were noted; but at Mach numbers greater than 0.8 camber provided either little improvement or had a detrimental effect on lift-drag ratio.

In general, the aerodynamic characteristics of the airfoils having the $a = 0.4$ mean line were found to be inferior to those of the $a = 1.0$ mean-line airfoils. John R. Spreiter, USA

1292. Bolz, Ray E., Note on interference between wing and body supersonic speeds, *J. aero. Sci.* **17,** 7, 453, July 1950.

Writer points out a mistake in equation (7) of C. Ferrari's paper on wing body interference [title source, **15,** 1948; AMR **1,** Rev. 1268]. An alternative solution is derived and the necessary subsequent modifications are indicated. The particular numerical results given by Ferrari are not affected by this correction.

It appears to reviewer that author's criticism is justified and that the second equation in (7) of Ferrari's paper does not in fact satisfy the relevant differential equation (4).

A. Robinson, England

1293. Numachi, Fukusaburo, and Fuchizawa, Sadatochi, **Theoretical research on profile-form of blade element suitable for arrangement in a straight grate** (in Japanese), *Rep. Inst. high speed Mech. Tohoku Univ.* **1,** 1–8, 9–15, 16–24; **2,** 77–84, 85–88, 101–105, 1949.

Authors present several methods of determining the shape of airfoils in a cascade so as to have a given pressure distribution. First four reports refer to the approximate treatment, in which solution is reduced to that of a solitary airfoil placed in a non-uniform stream, interference from the other airfoils being esti-

mated by replacing them by point-vortexes, flat-plate airfoils, or generalized Joukowsky airfoils. Flow around the airfoil is conformally transformed into flow around a circle, and coefficients in transformation function are determined by successive approximations. Last two reports refer to the more exact solution, in which flow past cascade of airfoils is transformed into flow around a circle. In each solution a numerical example is given, the prescribed pressure distribution being similar to that for the laminar-flow airfoil. Itiro Tani, Japan

1294. **Aiken, William S., Jr., and Howard, Donald A., A comparison of wing loads measured in flight on a fighter-type airplane by strain-gage and pressure-distribution methods,** *Nat. adv. Comm. Aero. tech. Note* 1967, 27 pp., 1949.

Pressure-distribution measurements were made on the wing of a fighter-type airplane to determine the span loading and to compare center-of-pressure results with those obtained by strain-gage measurements on the same airplane during a previous flight investigation. The flight tests were all made at a pressure altitude of about 30,000 ft and covered a Mach number range from approximately 0.35 to 0.81. Available wind-tunnel pressure-distribution data for a prototype of the test airplane are also included for comparison. Both flight and wind-tunnel pressure-distribution data are separated into additional and basic air-load components.

The agreement between shears, bending moments, and spanwise centers of pressure determined in flight from pressure-distribution data and strain-gage data was found to be good. During buffeting in low-speed stalls the spanwise center of pressure shifted farther outboard than during buffeting at Mach numbers near 0.80. From authors' summary

1295. **Horton, Elmer, A., Racisz, Stanley F., and Paradiso, Nicholas J., Investigation of boundary-layer control to improve the lift and drag characteristics of the NACA 65_2-415 airfoil section with double slotted and plain flaps,** *Nat. adv. Comm. Aero. tech. Note* 2149, 63 pp., Aug. 1950.

Boundary-layer suction was applied at the 45% chord station of the airfoil equipped with a double slotted flap and in vicinity of hinge line of airfoil with a deflected plain flap. The models were tested in the smooth and rough conditions at Reynolds numbers of 1.0×10^6, 2.2×10^6, and 6.0×10^6.

For same expenditure of suction power or for same flow coefficient, configuration with double-slotted flap gave higher maximum lift coefficients than did configuration with plain flap. Application of boundary-layer control at the 0.45 chord station of airfoil with double-slotted flap is seen to have little effect on linear portion of lift curve, whereas the data for the airfoil with plain flap show that boundary-layer removal causes large increase in lift for all angles of attack throughout the range investigated. Application of boundary-layer suction in vicinity of hinge line of airfoil with plain flap also increased the section lift-drag ratio for lift coefficients above 0.6 for rough condition and above 0.8 for smooth condition.

Maximum lift data are presented for an extensive range of airfoil thickness ratios. Julius Rotta, Germany

1296. **Young, A. D., and Booth, T. B., The profile drag of yawed wings of infinite span,** *Coll. Aero. Cranfield Rep.* 38, 15 pp., May 1950.

An approximate procedure for calculating the drag of infinite yawed wings is presented. The spanwise (parallel to leading edge) boundary-layer component is assumed to have the same profile shape as that for two-dimensional flow with zero pressure gradient and to have the same thickness as the local chordwise (perpendicular to leading edge) boundary-layer component. The single set of boundary-layer component measurements presented appear to substantiate the former but not the latter assumption. As is usual in the analysis of three-dimensional flows, any possible interaction between chordwise and spanwise turbulent velocity fluctuations is neglected so that simple sweep theory applies to the chordwise boundary-layer growth. These assumptions should permit simple and reasonable engineering estimates.
Gerald E. Nitzberg, USA

1297. **Brown, W. D., and Holford, J. F., Porous properties of various materials for use in making parachutes,** *Aero. Res. Coun. Curr. Papers*, C. P. 24, 9 pp., Mar. 1949, published 1950.

An investigation has been made into porous properties of paper perforated with round holes, and ribbon meshes with square interspaces. It has been established that the relationship between porosity and porous area is almost linear for all materials. An approximately straight-line law exists between the logarithm of porosity and the logarithm of pressure difference across sample being tested, whether that sample is a woven fabric or a ribbon mesh. The value of index x in expression $v \propto p^{1/2x}$ is lower for a ribbon mesh than for a woven fabric. Woven fabrics of natural fibers have a higher index value than woven fabrics of artificial fibers. From authors' summary

1298. **Andersson, Bengt, Theoretical studies of hydroplanes** (in Swedish), *Roy. Inst. Techn. Stockholm*, 76 pp., 1950.

Connection between a vessel's form and the pressure on surrounding water is theoretically treated. Main purpose is to find motion of the water and elevation of the surface around flat and speedy vessels. In main part of paper, Froude's number is supposed to be very great, i.e., influence of gravity and buoyancy are supposed to be very small. Then, a thin sheet of water is thrown out in free jets (the spray) ahead and to the sides of vessel. To make this clear, author gives an exact solution of problem for an infinitely long plate moving on water's surface in a direction perpendicular to length-direction of plate. Author suggests a device, a barring trin, to catch momentum of spray, thus reducing resistance against vessel's motion.

Having thus shown that the spray does not influence elevation of the water surface appreciably, author leaves the spray out of question. This simplification introduces other difficulties, viz., that the pressure turns out infinite at the fore edge of the wet surface of the planing hull. These infinities must fulfill certain conditions, which are found through a careful analysis.

At a given pressure on the wet surface of hull, the motion of water and elevation of the water surface is calculated by the aid of a Carleman's singular integral equation. The solution is carried through to the end in a special case. Instructions are given on how to perform the calculations for a planing hull with almost flat underside. O. H. Flaxen, Sweden

1299. **Dannenberg, R. E., Measurements of section characteristics of a 45° swept wing spanning a rectangular low-speed wind tunnel as affected by the tunnel walls,** *Nat. adv. Comm. Aero. tech. Note* 2160, 42 pp., Aug. 1950.

Two constant-chord wings, one unswept and the other swept 45°, that completely spanned a closed 7 × 10-ft wind tunnel, were investigated by pressure distribution and wake measurements, and tuft observations. Sections perpendicular to leading edge were the NACA 63_1-012. Angle of attack was varied from 0° to the stall for the unswept wing and from 0° to 12° for the swept wing. Equations are also presented for calculation of tunnel wall corrections for the swept wing.

Results indicate that pressure distribution and lift characteris-

tics over the central half of swept wing could be obtained with good accuracy from those of unswept wing by means of simple sweep theory. Small discrepancies were noted, however, in the drag and moment characteristics. John R. Spreiter, USA

1300. Spence, A., Effect of propeller thrust on downwash and velocity at tailplane. Data from low speed tunnel tests, *Aero. Res. Coun. Curr. Papers*, C. P. 21, 7 pp., Aug. 1947, published 1950.

Results of low-speed wind-tunnel model tests on longitudinal stability of five models have been analyzed to give effects of propeller thrust on angle of downwash and velocity at tailplane. Results give an indication of the magnitude of the effects and show some of the more important variables.

From author's summary

1301. Marx, A. J., An approximate method for the determination of the elevator deflections required for landing an aeroplane and of the extreme forward position of the center of gravity which is admissable in view of the landing characteristics, *Nat. Lucht-Lab. Amsterdam Rap.* V 1286, V19-V51, 1947.

Calculation of landing procedure (transition from steady glide to touchdown) is based on the three equations of longitudinal motion containing four variables, viz., speed, flight-path angle, angle of attack, and elevator deflection. In order to calculate the last one as a function of time during the nonstationary motion, a suitable relation between angle of attack and "relative" time (time in sec/time t_a required for flare out) is adopted. A similar function is established for the flight-path angle, the relation now containing a parameter p, which is introduced to adjust flight-path variation to chosen angle of attack variation. Then t_a, p, and the elevator deflection (as a function of time) can be determined from equations of motion. When elevator trim curves of the airplane are available for a few c.g. positions, the most forward c.g. position which can be tolerated is easily derived. It is shown how ground effect can be taken into account.

Although extent of calculations looks rather large, method can be applied in a reasonable time since the included auxiliary graphs facilitate computations. J. Buhrman, Holland

1302. Johnson, Harold S., and Haherman, John R., Wind-tunnel investigation at low speed of an unswept untapered semi-span wing of aspect ratio 3.13 equipped with various 25-percent-chord plain flaps, *Nat. adv. Comm. Aero. tech. Note* 2080, 27 pp., Apr. 1950.

Report presents the low-speed aerodynamic characteristics of an unswept untapered wing of NACA 64A010 section and aspect ratio 3.13 equipped with 25-per cent-chord unsealed plain flaps (deflected up to 60°) having various spans and spanwise locations. In general, changes in angle of attack, flap deflection, or flap spanwise location were found to produce trends in characteristics that were similar to, but of different magnitude from, those for unswept wings of higher aspect ratio.

Harvard Lomax, USA

1303. Smith, Hamilton A., and Schaefer, Raymond F., Aerodynamic characteristics at Reynolds numbers of 3.0 $\times$ 10^6 and 6.0 $\times$ 10^6 of three airfoil sections formed by cutting off various amounts from the rear portion of the NACA 0012 airfoil section, *Nat. adv. Comm. Aero. tech. Note* 2074, 18 pp., Apr. 1950.

Wind-tunnel tests of sections formed by removing 1.5, 4.0, and 12.5% of original airfoil. Resulting airfoils may be useful because of structural or control surface characteristics. Maximum lift coefficient changed less by cut-offs than by leading edge roughness. Drag coefficient increased; aerodynamic center moved rearward. H. Reese Ivey, USA

1304. Matheny, Cloyce E., Maximum pitching angular accelerations of airplanes measured in flight, *Nat. adv. Comm. Aero. tech. Note* 2103, 19 pp., May 1950.

Author derives simple empirical formulas relating gross weight and load factor to maximum pitching angular acceleration. Work is based on compilation of flight test data of conventional airplanes at Mach numbers below 0.8. A. Petroff, USA

1305. Taylor, J. Lockwood, Lift correction for finite chord, *Aircr. Engng.* **22,** 4, p. 174, June 1950.

An approximate correction for effect of finite aspect ratio A on lift curve slope is obtained from an analysis of the potential equation for a finite rectangular wing with uniform chordwise lift distribution by considering a mean value for loss of lift as compared to the two-dimensional case.

Reviewer notes that a simpler correction, which is even better for small aspect ratios, has been derived by N. Scholz (*J. aero. Sci.* **16,** p. 638, Oct. 1949) for the lift curve slope of elliptical plan forms as

$$2\pi[(1 + 4/A^2)^{1/2} + (2/A)]^{-1}$$

Scholz's equation has the correct limiting value for either large or small A of any plan form. E. V. Laitone, USA

1306. Mair, W. A., Research on high speed aerodynamics at the royal aircraft establishment from 1942 to 1945, *Aero. Res. Coun.* R. & M. 2222, 155 pp., 1950.

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 1019, 1325)

1307. Smith, F., The electronic simulator for the solution of flutter and vibration problems, *Aero. Res. Coun. Curr. Papers*, C. P. 26, 10 pp., Oct. 1949, published 1950.

Author discusses the basic computing properties of various elements of an electronic analog of flutter and vibration problems. A circuit diagram showing a combination of these elements for simulating a one-degree-of-freedom flutter equation is given. A block diagram for solving a six-degree of freedom case is also shown. The R.A.E. has started work on the design of such a six-degree-of-freedom simulator, and in the meantime a two-degree of freedom prototype is being made.

The flutter equations presented in paper are of the form

$$A_{11}\ddot{x}_1 + (B_{11}v + D_{11})\dot{x}_1 + (C_{11}v^2 + E_{11})x_1 + A_{12}\ddot{x}_2 + + + = 0$$

where v is airspeed and A, B, C, D, E, are constants. Author remarks that this is customary practice in England, but fails to state that this implies no change in Theodorsen's $C(k)$ function with reduced speed. Walt Targoff, USA

1308. Schindler, A., Essay on justification of a dynamic determination of frequencies and associated characteristic functions of an airplane and of a calculation of the corresponding generalized masses (in French), *Rech. aéro. Paris*, no. 15, 15–18, May-June 1950.

Paper concerns basic theory underlying the excitation of normal modes by means of multiple shakers of electrodynamic type. A formula is presented for the generalized mass associated with a normal mode, utilizing only vibration test data and not requiring

a determination of weight distribution. Details of testing technique and equipment design are not discussed.

M. J. Turner, USA

1309. **Barmby, J. G., Cunningham, H. J., and Garrick, I. E., Study of effects of sweep on the flutter of cantilever wings,** *Nat. adv. Comm. Aero. tech. Note* 2121, 74 pp., June 1950.

Paper presents results of many systematic wind-tunnel tests to study variations in mass parameters, tip shape, aspect ratio, Mach number (up to 0.85), etc. Although precise separation of individual effects was not achieved, data show order of magnitude of some combined effects and should prove useful to workers in this field. All tests were limited to cantilever models. Theoretical method is presented which takes sweep into account but does not treat aspect ratio or compressibility. Check of theory using only first bending and torsion uncoupled modes shows method satisfactorily predicts main effects of sweep at least for nearly uniform cantilever wings of moderate length to chord ratios and Mach numbers up to 0.8. Need for additional theoretical and experimental studies is indicated. B. Smilg, USA

1310. **Huckel, Vera, and Durling, Barbara J., Tables of wing-aileron coefficients of oscillating air forces for two-dimensional supersonic flow,** *Nat. adv. Comm. Aero. tech. Note* 2055, 73 pp., Mar. 1950.

Paper contains an extensive tabulation of oscillating air forces for wings with flaps in two-dimensional supersonic flow. These coefficients are listed for Mach numbers of 10/9, 5/4, 10/7, 5/3, 2, and 5/2; for ratios of flap chord to wing chord of 0.1 to 0.9 at 0.1 interval; and for frequency parameters ($\bar{\omega}$) from 0.02 to 20.0 at various intervals. Reviewer believes that numerical values in these tables may be made more uniform if they are multiplied by k, k being the reduced frequency. C. C. Wan, USA

1311. **Weber, R., Tables of unsteady aerodynamic coefficients for two-dimensional supersonic flow. III. Graphical representation of unsteady aerodynamic coefficients and tables of Küssner-Jordan** (in French), *Off. nat. Étud. Rech. aéro. Rep.*, publ. 41, 134 pp., Dec. 1950.

See AMR **4**, Rev. 809.

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1220, 1279, 1330, 1362)

1312. **Richard, G. C., Definitions of the principle of the helicoplane** (in French), *Rech. Aéro.*, no. 15, 3–7, May-June 1950.

Author believes that he has an original solution to problem of providing an aircraft to combine the qualities of airplane and helicopter. This solution, first proposed some twenty years ago, is called the "helicoplane" and comprises one or more rotors with their axes nearly horizontal and with blades undergoing a cyclic-pitch change. Photographs are given of a model rotor on a test stand and reference is made to wind-tunnel tests, although no performance graphs or data are included.

It is pointed out as a remarkable fact that the lift of a propeller when operating with its axis at a small positive angle to the horizontal will be many times the product of the thrust and the sine of the angle; but this is a phenomenon which is analyzed in generally available reports, such as Ribner's [*NACA Rep.* 819, 1945], on the side force of a propeller. Author is very sketchy in his discussion of parallel work done in other countries on the problem of designing a vertically rising airplane and gives no references to articles published outside France.

Great claims are made for the helicoplane, including the possibility of vertical flight (and this without necessarily tilting the rotor axis from the horizontal), but article does not seem to prove all the claims. W. C. Johnson, Jr., USA

1313. **Legras, J., The optimum helicoplane** (in French), *Rech. Aéro.*, no. 15, 9–14, May-June 1950.

The helicoplane is a propeller whose blade angle varies periodically, so that, in addition to the usual thrust and torque, a force in the plane of the propeller may be produced. Let this force be decomposed with respect to a fixed set of axes into components called lift and side force. Author calls an optimum helicoplane one for which the induced power loss (as usually defined for propellers) is minimum for given lift and torque. This problem is attacked here by more or less conventional methods involving helicoidal trailing vortex sheets, and a few results are obtained.

W. R. Sears, USA

1314. **Markov, N. M., On the spatial flow in consecutively situated guiding and rotating turbine grids** (in Russian), *Doklady Akad. Nauk SSSR* **71**, 2, 245–248, Mar. 1950.

This short note indicates the derivation of an equation necessary for analysis of radial flow of water in water wheels and for calculation of the boundary layer. Analysis of equations confirms the possibility of deviation of flow not only toward tip of blades but also toward their roots. L. M. Tichvinsky, USA

1315. **Stanitz, John D., Two-dimensional compressible flow in turbomachines with conic flow surfaces,** *Nat. adv. Comm. Aero. Rep.* 935, 25 pp. 1949.

Application of the "relaxtion method" of Southwell to the determination of compressible flow in turbomachines on hypothesis that stream surfaces of flow are right circular cones with axes coincidental with that of machine. For an easier application of the method, author makes use of a conformal transformation with which flow surface is represented in a plane, so that blades of arbitrary shape become thin, straight, and parallel in transformed plane. Results are reported of calculations in the case of a centrifugal compressor, presuming both a compressible fluid and an incompressible fluid. Comparison reveals a large compressibility effect upon streamline configuration.

Carlo Ferrari, Italy

1316. **Wu, Chung-Hua, and Wolfenstein, Lincoln, Application of radial-equilibrium condition to axial-flow compressor and turbine design,** *Nat. adv. Comm. Aero. Rep.* 955, 30 pp., 1950.

See AMR **2**, Rev. 920.

1317. **Wu, Chung-Hua, Sinnette, John T., Jr., and Forrette, Robert E., Theoretical effect of inlet hub-tip-radius ratio and design specific mass flow on design performance of axial-flow compressors,** *Nat. adv. Comm. Aero. tech. Note* 2068, 33 pp., Apr. 1950.

An analysis is made for multistage axial flow compressors designed on basis of approximately symmetrical velocity diagrams at all radii, constant total enthalpy along the radius, a maximum Mach number of 0.8 for all stages, and a prescribed empirical turning limitation. Analysis covers inlet hub-tip-radius ratios of 0.6, 0.5, and 0.4, for practical range of specific mass flows for each, and the number of stages sufficient to give an exit hub-tip ratio of approximately 0.9. Method of calculation is essentially based on the simplified radial-equilibrium calculation of *NACA Rep.* 955 (see AMR **2**, Rev. 920).

For any given inlet hub-tip radius ratio the design rotor speed and the pressure ratio per stage decreases with increasing mass

flow per unit frontal area. The maximum pressure ratio and rotor speed decreases with decreasing inlet hub-tip-radius ratio. The specific power input per stage is fairly constant over the practical range of mass flows and varies only slightly with inlet hub-tip-radius ratio. The analysis is particularly helpful in evaluating axial-flow compressor designs, within the given limitations, for either maximum mass flow with minimum frontal area or minimum number of stages. H. E. Sheets, USA

1318. Finger, Harold B., Method of experimentally determining radial distributions of velocity through axial-flow compressor, *Nat. adv. Comm. Aero. tech. Note* 2059, 25 pp., Apr. 1950.

A method is presented for determining velocity distributions downstream of any compressor blade row if the outlet total temperatures, total pressures, and relative flow angles are known. Method has been applied to determination of velocities and pressures in a 10-stage compressor and has indicated good agreement with measured wall-static pressures.

From author's summary by H. E. Sheets, USA

1319. Bammert, K., and Klaukens, H., Hub dead water behind the guide wheels of axial flow machines (in German), *Ingen.-Arch.* **17,** 367–390, 1949.

Fluid motion in a duct limited by two coaxial cylinders is studied, having given a preliminary torsion to the fluid through fixed wheels. Hypotheses: permanent motion, axial symmetry, ideal incompressible fluid. Through the wheels the motion is assumed to be isoenergetic, over cylindrical stream surfaces. Behind them this assumption is no longer valid, because of presence (eventual) of a hub region of dead water. Radius of that region is found as a function of the ratio: pressure/(mass flow)^2, and the ratio of the peripheral to axial velocity, in both cases, of free-vortex flow and of straight blades. Minimal R_{hub}/R_{tip} ratios to prevent any dead water formation are determined (in all cases the result is less than 0.6); motion in duct and at its exit is studied in detail. Theoretical and experimental results are compared.

Gino Moretti, Argentina

1320. Erwin, John R., and Emery, James C., Effect of tunnel configuration and testing technique on cascade performance, *Nat. adv. Comm. Aero. tech. Note* 2028, 55 pp., Feb. 1950.

Investigation to determine influence of aspect ratio, boundary-layer control on walls, Reynolds number, and tunnel end-wall conditions upon performance of airfoils in cascade at low speeds. Two-dimensional flow was not realized for any compressor cascade configuration with solid walls, and large discrepancies in results were noted, casting doubt upon results of other investigators, e.g. [*NACA A.C.R.* L5F07a, 1945, and *NACA T.N.* 1271, 1947]. Discrepancies attributed to interference and interaction of boundary layers on side walls with flow over cascade airfoils. No similar difficulties found for turbine cascades. Careful control of boundary layer by means of slots and porous surfaces led to results satisfying two-dimensional flow criteria.

J. M. Wild, USA

1321. Hutton, S. P., Thin aerofoil theory and the application of analogous methods to the design of Kaplan turbine blades, and discussion, *Proc. Inst. mech. Eng. Hydraul. Group* (*W.E.B. no.* 57), **163,** 81–97, 1950.

Basic problem, reviewed in elementary fashion, is design of cascade airfoils with low, reasonably constant, or prescribed perturbation velocities for minimization of cavitation effects with high lift. Author advocates using recent methods of designing cascade airfoils with prescribed velocity distribution, though, admittedly, velocity distribution to be designed into actual machine for high performance is unknown. He argues that considerable gains could be made in turbine performance by such methods of blade design.

In discussions and communications which follow paper, issue is taken with author for some of his conclusions; general theme of this section (with which reviewer agrees) is that it is doubtful if author's statements, obtained from two-dimensional perfect fluid and single operating point considerations, have many immediate applications in the design of turbomachines. It is felt that performance of "conventional" blades would be comparable, under actual operating conditions, with that of the author's "ideal" shapes. B. Augenstein, USA

1322. Shames, Harold, Himmel, Seymour C., and Blivas, Darnold, Frequency response of positive-displacement variable-stroke fuel pump, *Nat. adv. Comm. Aero. tech. Note* 2109, 32 pp., June 1950.

One type of investigation of transient operation of jet power plants for airplanes involves a knowledge of frequency response characteristics of fuel pumps. Response characteristics for sinusoidal excitation are reported for one wobble plate type pump where outlet fuel pressure is a linear function of control oil pressure produced by a hydraulic sine wave generator of variable amplitude, frequency (0 to 10 cycles per sec) and system pressure. The pump approximated a first-order lag system with time constant about 0.04 sec. Response is independent of input amplitude and fuel flow rates but varies with pump speed (time constant decreased as speed increased). Pump lag is small compared to lag of present jet engines. Curves of significant pump performance items are included.

Richard G. Folsom, USA

1323. Lundin, Bruce T., Theoretical analysis of various thrust-augmentation cycles for turbojet engines, *Nat. adv. Comm. Aero. tech. Note* 2083, 52 pp., May 1950.

The results of analytical studies of tail-pipe burning, water injection and bleed-off methods of thrust augmentation are presented which provide an insight into the operating characteristics and summarize the performance that may be obtained, when applied to a typical turbojet engine.

To obtain a high thrust augmentation and to minimize the loss during nonburning, the pressure losses (velocities) in the tail pipe equipped with afterburners should be low. Thrust-augmentation ratios are about 1.5 at Mach number 0, and 3.0 at Mach number 2.0. Specific fuel consumption figures are about 2.5.

Water injection is considered both for injection into the compressor inlet and into the combustion chambers. With air at the compression outlet saturated (injection into the inlet), the thrust-augmentation ratio varies from 1.4 at Mach number 0 to 2.6 at Mach number 2.0. Specific liquid consumption is about 3.0 at Mach number 0 and at sea level. It increases rapidly with Mach number. Water injection into the combustion chambers shows augmentation ratios that are slightly less than those of injection into the compressor inlet. Specific liquid consumptions are roughly 2 times as great.

Thrust augmentation by means of bleed-off is the highest of the three methods (2.3 at Mach number 0). Specific liquid consumption, however, is very high. J. G. Slotboom, Holland

1324. Feder, Melvin S., and Hood, Richard, Analysis for control application of dynamic characteristics of turbojet engine with tail-pipes burning, *Nat. adv. Comm. Aero. tech. Note* 2183, 39 pp., Sept. 1950.

Dynamic behavior of a turbojet engine with tail-pipe burning is studied to aid in synthesis of the engine control system. Engine

is considered to be a linear system in which incremental changes from steady-state operating conditions are considered and general transfer functions for this engine configuration are derived. The dominant dynamic characteristics of this engine, determined from steady-state engine data and one transient relation, can be used in basic control design.

Application of analysis to scheduled controls results in a relation between tail-pipe-burner fuel flow and exhaust-nozzle area which maintains constant engine speed and temperature over a range of afterburner operation. A relation between engine fuel flow and turbine outlet temperature which maintains constant engine speed over a range of exhaust-nozzle areas is also presented. For noninteraction controls, analysis yields the form of all the control functions, and a solution for six out of a possible seven required control functions in terms of steady-state engine data.

Simon Ostrach, USA

1325. **Russell, A. E., Some factors affecting large transport aeroplanes with turboprop engines,** *J. aero. Sci.* **17,** 2, 67–106, Feb. 1950.

Main part of paper, the 13th Wright Brothers Lecture, concerns itself with the technical problems that arise in design of large airplanes. Subsidiary sections concern themselves with characteristics of turboprop engines and aerodynamic operating efficiency. The high lights of paper may be tabulated as follows: 1. Turbojet and turboprop give equal performance at medium ranges and divide the field otherwise for short and long ranges, respectively. Turboprop offers greater economy at higher cruising speed and higher altitude. It also has advantages with respect to noise, mechanical simplicity, reliability, and the lesser power output restrictions of rotating over reciprocating machines. 2. In a discussion of relative importance of engine parameters, those considered are: Compression ratio, peak temperature, compressor efficiency, turbine efficiency, ratio of prop to jet power, and engine scale. 3. Influence of following airplane performance characteristics are explored: Cruise plan, initial weight, ambient temperature, engine failure, wing design, and power loading. Flight at constant potential rate of climb is the optimum of the four plans considered. Potential pay load as well as fuel load are approximately a fixed percentage of gross weight. Ambient temperatures have a greater effect on turboprops than on reciprocating engines, and it appears that cruise at constant incidence is optimum from the standpoint of temperature effect. Failure of a power unit is likened to temperature rise effect. No optimum wing loading is apparent from standpoint of pay load and range performance. Range is favorably affected by increasing aspect ratio. Power loading is not a significant range or pay-load parameter. 4. New structural problems arise as a result of large scale. Normal mode analysis appears to be most reliable. Conventional methods of attack, that is calculation, simulation, and full scale tests are to be used. Method of calculation is a trial and error iteration. Automatic computing machines enable more detailed study. As in the calculations, model simulation complications increase out of direct proportion to information sought. Thus, detail model method is not suitable for routine design analysis. In resonance full scale tests support of the prototype is critical. Of the several methods of indicating or recording measurements, it appears that continuous multichannel film recording is optimum for accuracy and detail. Flight testing is practical only for the lower vibration modes and has the advantages of actual flight loading. On the question of structural dynamic response, it is found in practice that dynamic loadings are small for high frequency modes. 5. Gust loads are affected by several time lags in coupled aerodynamic and structural responses. One must either fall back on experience to allow for these effects or make estimates from test data taken under conditions which greatly reduce coupled effects. For large airplanes importance of gusts diminishes with increasing cruising speed. Gust dynamic wing loading reaches a maximum (presently at 250 to 300 mph) and then sharply decreases with increasing speed. This speed maximum decreases with increasing airplane size, and gust loadings increase with increase in altitude. Gust alleviation through wing flexure and aileron control are described. Both present flutter hazard, but latter shows by far the greater promise of freedom from flutter. 6. On flutter questions, model testing is suitable only to obtain basic flutter data. Calculations must do the rest and here the analog and digital computing machines are of great value. A wing flexure-torsion analysis is discussed to show how optimum wing loadings from the standpoint of wind weight may be estimated. 7. A progressive attack is made on wing-layout problem which goes successively from the general intuitive know-how study to detailed fabrication and weighing of components. Significant points here are: (a) Without gust alleviation, wing loading does not influence pay load and is selected by other factors, such as take-off run, and flying with disabled engines; (b) with gust alleviation, moderate wing loading gives higher pay loads; (c) aspect ratios in excess of twelve to fifteen are profitable goals. 8. On fuselage design, single deck is optimum for long range while double deck is optimum for high load, short range condition, and also for volumetric pay loads. 9. Landing gears of wheel groups reduce substantially runway loading without increase in weight cost, and thus larger planes do not need specially constructed runways at prohibitive cost.

M. G. Scherberg, USA

Flow and Flight Test Techniques

(See also Revs. 1243, 1259, 1286, 1294, 1299, 1302, 1309, 1320, 1365, 1376)

1326. **Zalovcik, John A., A method of calibrating airspeed installations on airplanes at transonic and supersonic speeds by use of accelerometer and attitude-angle measurements,** *Nat. adv. Comm. Aero. tech. Note* 2099, 44 pp., May 1950.

A method is described for calibrating airspeed installations on airplanes at transonic and supersonic speeds in vertical plane maneuvers. Measurements of normal and longitudinal accelerations and attitude angle are used. Method involves starting or ending a calibration run near level flight at a speed for which airspeed calibration is known and for which free-stream static pressure may be determined. Integration of the vertical acceleration computed from normal and longitudinal accelerations and the attitude angle determines the change in altitude which, when combined with the temperature measurements, gives the change in static pressure from the start or end of the calibration run and hence the variation of free-stream static pressure during the calibration run. The static-pressure error is then obtained at any instant during the calibration run by subtracting the free-steam static pressure from the indicated static pressure. In this method the required instrumentation is carried within the airplane. A study of the effect of various sources of error indicated that the required quantities can be measured accurately enough to insure a satisfactory calibration.

From author's concluding remarks by Byron E. Short, USA

1327. **Zalovcik, John A., A radar method of calibrating airspeed installations on airplanes in maneuvers at high altitudes and at transonic and supersonic speeds,** *Nat. adv. Comm. Aero. Rep.* 985, 7 pp., 1950.

See AMR 3, Rev. 1573.

1328. Bidwell, Jerold M., Analysis of an induction blowdown supersonic tunnel, *Nat. adv. Comm. Aero. tech. Note* 2040, 28 pp., Apr. 1950.

Running time of one type of induction blowdown supersonic tunnel is studied on the basis of certain assumptions regarding flow conditions and pressure losses. Calculations are carried out for test-section Mach numbers from 1 to 2 and reservoir pressures from 2 to 4 atm, though the basic equations are of general validity. Charts are presented showing area and static pressure (or stagnation temperature) which the inducing jet must have to produce maximum running time when size and conditions of reservoir and test section are specified.

Results indicate that, with optimum inducing jet, running time of a tunnel having a given area of test section and given initial conditions in reservoir increases with increasing test-section Mach number until the Mach number reaches 1.35, after which the running time decreases rapidly. Comparison of induction tunnel with a direct-discharge tunnel of equal size shows that, over most of the specified range of Mach numbers and reservoir pressures, the induction tunnel provides much the greater running time. Comparison of induction tunnel with a direct-discharge tunnel of different test-section area but equal running time shows that induction tunnel then provides the greater Reynolds number.

Walter G. Vincenti, USA

1329. Tournier, M., On the utilization of high frequency current for heating of hot wire anemometers (in French), *Rech. Aéro.* no. 15, 19–25, May-June 1950.

A hot-wire anemometer for flat response from zero to 3000 cps frequency is described, the filament of which is heated by a current of high frequency of the order of 100 kc. The filament, placed in a Wheatstone bridge, is kept at nearly constant temperature by an amplifier and current regulator controlled from the unbalance of the bridge. Automatic stabilization of the equilibrium of the bridge is achieved since cooling of filament will unbalance the bridge, and this in turn will increase heating current, thereby reestablishing nearly the original filament temperature.

A tuned input circuit placed in series with bridge ahead of the first amplifier tube will increase signal of bridge approximately 50 times, without increasing random noise so that the noise-to-signal ratio of instrument is reduced to $^1/_{50}$ as compared with direct input. This enhances the accuracy of investigations of phenomena of turbulence of low intensity.

Gains of amplifier tubes at high frequency of 1000 may readily be obtained as compared with 200 at low frequencies, hence an over-all gain of 50,000 per stage is possible. An auxiliary corrective amplifier stage is devised by which attenuation is suppressed as a function of the modulation frequency to increase accuracy of measurement of upper range of frequency spectrum of turbulence.

The spectrum of turbulence may be analyzed directly, without the necessity of measurements of time correlations, by a circuit consisting of a frequency-changer stage, detector, and a highly selective band filter, using as a standard reference a quartz crystal oscillator.

To determine spatial correlation coefficients, two distinct hot-wire filaments heated by high frequency currents of equal or different frequency are used, the signal from which, after detection, is impressed upon circuit elements which yield mean values of the required products of instantaneous values. The high frequency carrier principle employed conserves the low frequency components of the spectrum and permits the measurement of permanent changes of velocity. J. R. Weske, USA

1330. Koenig, R. J., and Cesaro, Richard S., Investigation of spark-over voltage-density relation for gas-temperature sensing, *Nat. adv. Comm. Aero. tech. Note* 2090, 27 pp., May 1950.

Some literature about the spark-over voltage-density relation is reviewed (in formula 4, E and T must be changed) and measurements of this relation in air for several gap spacings and the effects of electrode-surface finish, gas velocity, and of a static pressure tap in one of the electrodes are reported. From pressure and density, temperature of the gas is known. Developed temperature-sensing probe can be used, e.g., in a gas-turbine engine. H. Wijker, Holland

1331. Neihouse, Anshal I., and Pepoon, Philip W., Dynamic similitude between a model and a full scale body for model investigation at full-scale Mach number, *Nat. adv. Comm. Aero. tech. Note* 2062, 8 pp., Mar. 1950.

Report presents an analysis and a set of formulas for transcribing data from dynamic flight tests at full-scale Mach number of a scaled-down model, representing some configuration such as an aircraft, pilot escape capsule bomb, etc., to corresponding performance data of a full scale unit. Effect of the gravity force is compromised for this type of simulation, but authors state that error is relatively small for short time durations of flight. No data are exhibited to indicate a quantitative measure of this error.

M. G. Scherberg, USA

1332. Burwell, Robert L., Jr., Metlay, Max, and Pfohl, Frank W., The measurement of rates of flow of gases by wet test meters, *Rev. sci. Instrum.* **21,** 8, 681–682, Aug. 1950.

Rates of flow of gases may be measured to an accuracy of better than 0.1% with properly modified and operated wet test meters. The period of single rotation is measured by an electric stop clock. Instrument is not an absolute one but must be calibrated by a meter prover. A suitable prover is described. Within rather wide limits the calibration is independent of flow rate. Instrument may be used for direct measurement or for calibrating other types of flowmeters. From authors' summary

1333. Scherer, Michel, Wind-tunnel measurements of aerodynamic derivatives of an aircraft model by the method of analysis of displacements (in French), *C. R. Acad. Sci. Paris* 230, 26, 2260–2262, June 26, 1950.

The supporting system for the model is arranged to give six degrees of freedom, so that displacements in any one of these can be measured independently of the others. Model is restrained by springs of known stiffness. An oscillating force or couple is applied and resulting displacements in one or more degrees of freedom are recorded on oscillographs. If model is made to satisfy conditions for dynamical similarity, the various stability derivatives can be calculated from recorded displacements. Some numerical values obtained by this method on one model are given. Apparatus and methods are not described in detail.

W. A. Mair, England

1334. Taylor, Marion K., A balsa-dust technique for air-flow visualization and its application to flow through model helicopter rotors in static thrust, *Nat. adv. Comm. Aero. tech. Note* 2220, 47 pp., Nov. 1950.

A method for air-flow visualization by finely divided particles of balsa wood introduced into the air is described. Photographs show its application on coaxial and biaxial model helicopter rotors operating in static thrust in both steady-state and transient-flow conditions.

Though good results are obtained, the application of a number of hot-wire oil vaporizers, as developed by the Aeronautical Re-

search Institute of Holland for wind-tunnel tests and as used for investigation of model helicopter rotors, seems to have some advantages, e.g., better separation of streamlines, clearer pictures of the flow pattern easy to analyze, cleaner operation especially in closed circuit wind tunnels, and controllability of amount and intensity of smoke threads. T. V. Oosterom, Holland

1335. **Kane, E. D., and Maslach, G. J., Impact-pressure interpretation in a rarefied gas at supersonic speeds,** *Nat. adv. Comm. Aero. tech. Note* 2210, 37 pp., Oct. 1950.

Authors used source-shaped impact tubes and conical static tube to determine the flow Mach number of a rarefied gas in a supersonic tunnel in order to check the validity of the Rayleigh Pitot-tube equation.

Comparative results indicate that impact pressure for a given free-stream Mach number and static pressure as determined from the Rayleigh relation yield an impact pressure which is lower than the measured value. A boundary-layer theory and kinetic theory analysis for the impact pressure tube are also reviewed.

Present work is a first step in applying the impact tube for flow measurements in wind tunnels producing rarefied gas flows. The impact tube will become more useful when it can be checked against a primary standard for determining the Mach number of a rarefied gas. Irving I. Glass, Canada

Thermodynamics

(See also Revs. 1241, 1263, 1324, 1353, 1367)

1336. **Goff, J. A., and Gratch, Serge, Zero-pressure thermodynamic properties of carbon monoxide and nitrogen,** *Trans. Amer. Soc. mech. Engrs.* 72, 6, 741–749, Aug. 1950.

Tables are represented of principal thermodynamic properties of CO and N_2 along the zero-pressure isobar in range 100 to 5000 deg R at intervals indicated by the number in brackets as follows: 100 [5] 130 [10] 240 [20] 500 [50] 1000 [100] 2000 [200] 5000. Data have been computed by method of quantum statistical mechanics in the University of Pennsylvania Thermodynamics Research Laboratory. It is believed that tables given should supersede all previous ones. Calculated values are in satisfactory agreement with those obtained from calorimetric and acoustic velocity measurements. K. W. Taconis, Holland

1337. **MacCormack, K. E., and Schneider, W. G., Compressibility of gases at high temperatures. IV. Carbon dioxide in the temperature range 0°–600° C, and pressures up to 50 atmospheres,** *J. chem. Phys.* 18, 9, 1269–1272, Sept. 1950.

The compressibility of carbon dioxide has been measured in the temperature range 0–600 C, at pressures up to 50 atm, by a method employing gas expansion. Data are presented for the virial coefficients of the series equation, PV (Amagat units) = $A_t + B_tP + C_tP_1^2 + D_tP_1^4 + \ldots$ and values of PV (Amagat) at integral values of pressure and temperature.

From authors' summary

1338. **MacCormack, K. E., and Schneider, W. G., Thermodynamic properties of gaseous carbon dioxide at temperatures from 0–600° C, and pressures up to 50 atmospheres,** *J. chem. Phys.* 18, 9, 1273–1275, Sept. 1950.

Compressibility data presented in a previous paper have been used in conjunction with spectroscopically determined zero-pressure specific heat data for CO_2, to calculate thermodynamic properties over the temperature range 0–600 C at pressures up to 50 atm.

From authors' summary

1339. **Michels, A., Wassenaar, T., Zwietering, Th., and Smits, P., The vapour pressure of liquid carbon dioxide,** *Physica* 16, 5, 501–504, May 1950.

Measurements were made from −56 C to +3 C, and data can be expressed as $\log_{10}P = -1353.202/T - 8.142537 \log_{10} T + 0.0062591567T + 2.461930$, where P = pressure, atmospheres; T = temperature, K. Special attention was given to measurements at 0 C. R. E. Treybal, USA

1340. **Shorin, S. N., Role of radiation energy in combustion processes** (in Russian), *Izvestiya*, no. 7, 995–1015, July 1950.

By comparing distribution of radiation and of kinetic energies between carriers in an elemental volume, author shows that the high level energy photons have a relatively larger statistical weight than gas molecules having same energy. Also, under corresponding conditions, probability of photon activation of combustible mixture can be considerably higher than probability of activation by collision of molecules. This is due to radiation absorption of admixtures in combustible mixtures.

It is shown analytically that, during heating-up process of the combustible mixture before its ignition, major role is due to heat of reactions induced by photon activation in incoming combustible mixture. On basis of general energy equation, it is possible to show radiation behavior during combustion processes at various conditions.

An equation is derived to determine the velocity of flame propagation by photon activation of molecules, which, by its character of variation of different parameters, fully corresponds to available experimental material. L. M. Tichvinsky, USA

1341. **Turner, L. Richard, and Bogart, Donald, Constant-pressure combustion charts including effects of diluent addition,** *Nat. adv. Comm. Aero. Rep.* 937, 24 pp., 1949.

To aid in the calculation of thermodynamic data for jet engine cycles, a series of 15 graphs is presented from which the final temperature of constant pressure combustion of hydrocarbon fuels and the amount of fuel required to attain a given temperature of combustion are obtained. These graphs are primarily designed to take into account the effect of adding combustible diluents to the air of combustion.

The following liquid diluents are considered: N_2, O_2, CO_2, NH_3, and the alcohols $(CH_2)_nH_2O$ where $n = 0$ (water), 1, 2, and 3. Further parameters included in charts are: Moisture in air, incomplete combustion of diluent and fuel, air temperature, and reheating of combustion gases. Dissociation of combustion gases is not taken into consideration. Effect of each of these parameters on combustion temperature and amount of fuel is expressed by a corresponding correction factor which is obtained from individual graphs or alignment charts, the use of which is illustrated by examples. E. F. Lype, USA

1342. **Povolny, John H., Bogdan, Louis J., and Chelko, Louis J., Cylinderhead temperatures and coolant heat rejection of a multicylinder liquid-cooled engine of 1650-cubic-inch displacement,** *Nat. adv. Comm. Aero. tech. Note* 2069, 63 pp., Apr. 1950.

The cooling characteristics of a 1650-cu in. liquid-cooled aircraft engine were measured by means of a bench test. Testing arrangement is described. Plots of the cylinder-head temperature (measured between the exhaust valves) and coolant heat rejection as functions of the principal engine and coolant variables are shown. Both head temperature and coolant heat rejection increase with charge flow, inlet manifold temperature, leaning of the fuel-air ratio from 0.115 to 0.067, exhaust pressure, and ignition timing in excess of 36 deg B.T.C. An increase in cylinder-head temperature with decreasing heat rejection results after

increasing the ethylene-glycol content of the coolant. All data were correlated by the NACA correlation method and compared with previous tests on a different engine. Except for the points where boiling of the coolant occurred, agreement between both test series is good. E. Haenni, Switzerland

1343. Clark, J. S., Combustion in aero gas turbines, *Engineering* **170,** 4416, 230–232, Sept. 1950.

A brief but comprehensive review is presented of the major combustor design features and factors affecting the choice of design. The most successful design represents a compromise between the ideals of high efficiency stability, simplicity, and reliability on the one hand, and low pressure loss, size, and weight on the other. Combustors are operating successfully with combustion times of 0.01 sec, heat release rates of 5.4 Btu/(hr)(ft^3)(atm), and with pressure losses of only 3–5% of the combustor-inlet total pressure.

The over-all process is complex and composed of various individual processes, namely, fuel atomization, fuel vaporization, mixing of fuel and air, and combustion of the fuel. Each of these individual processes is briefly discussed from the standpoint of theory and the engineering techniques currently practiced.

J. Howard Childs, USA

1344. Manes, Milton, Hofer, L. J. E., and Weller, Sol, Classical thermodynamics and reaction rates close to equilibrium, *J. chem. Phys.* **18,** 10, 1355–1361, Oct. 1950.

Generally accepted concepts are used to show that, in general, the net reaction rate of a reversible reaction at a condition close to equilibrium is directly proportional to the change in value of the governing thermodynamic function per unit amount of reaction and, specifically, the net rate near an equilibrium under a constant total pressure is proportional to the Gibbs free energy change ΔF. By means of a further plausible assumption it is further deduced that the relationship is frequently shown by the expression $r_f - r_b = -z \cdot r_f(\Delta F/RT)$, where r_f and r_b are the rates of the forward and backward reactions, ΔF, R, and T have the usual meanings of free energy change, gas constant, and absolute temperature, and z is an experimentally determined constant. Such relationships are independent of the way in which any displacement from equilibrium arises. Hence, it is possible to determine the relative effect of change in concentration of any specified reactant on the gross rate of the forward (or backward) reaction by measuring the net reaction rate for a series of reaction mixtures whose equilibrium concentrations of the specified reaction vary widely. The order of the gross reaction with respect to the specified reactant may be estimated from data similar to those obtained by already accepted methods.

Authors do not report any experimental tests of this method of investigation which requires a precise analytic technique and does not yield data different from those of the classical methods. It appears to be useful, primarily, for reactions between high temperature gases, and possibly between ions in solution which are too rapid for convenient study by classical methods.

Jack D. Bush, USA

1345. Huff, Vearl N., and Morrell, Virginia E., General method for computation of equilibrium composition and temperature of chemical reactions, *Nat. adv. Comm. Aero. tech. Note* 2113, 47 pp., June 1950.

By this method of successive approximations one can determine simultaneously equilibrium composition and temperature or the composition of an assigned temperature of a system having a large number of reactants. Assuming ideal gases and starting with estimates of composition and temperature, a set of linear correction equations is set up in matrix form; their solution furnishes new values of composition and temperature to be used in the next approximation. Equations are derived and examples given for combustion at constant pressure or volume and for isentropic expansion. The method appears to be rapidly convergent and simple to use. A. Fejer, USA

1346. Macintire, H. J., and Hutchinson, F. W., Refrigeration and engineering, 2nd ed., New York, John Wiley & Sons, Inc., 1950, x + 610 pp. $6.50.

Part I of book gives elementary, but practical, discussion of the thermodynamics of refrigeration. Part II discusses load determination, and part III is a general, descriptive section on refrigeration equipment.

Chief merit of book is authors' comprehensive treatment of heat-transfer phenomena in refrigeration, both steady state and transient. Graphical solutions for film coefficients of heat transfer are convenient for rapid approximations. Heat pump material contains transient heat-flow problems connected with energy sources and energy sinks, but gives only short discussion of pipe grids, the most common form of earth heat exchanger.

Authors correctly state that fluid properties of Nusselt's general equation vary with temperature, but do not say that these properties may significantly change if the fluid stream temperature differs appreciably from estimated mean film temperature.

Since air distribution is vital to certain refrigeration applications, the section on fluid flow and duct design is too brief and general, in reviewer's opinion. Harrison D. Goodman, USA

1347. Wilke, C., A viscosity equation for gas mixtures, *J. chem. Phys.* **18,** 4, 517–519, Apr. 1950.

Prediction of viscosity of gas mixtures from knowledge of properties of the components is obtained by a semi-empirical expression. Formula is tested by comparison with data on 17 binary mixtures and 7 multicomponent mixtures (e.g., combustion products). Agreement is satisfactory (average deviation less than 2%) for most design purposes. E. D. Kane, USA

1348. Knudsen, Martin, The kinetic theory of gases, 3rd ed., New York, John Wiley & Sons, Inc., 1950, vii + 64 pp., 20 diagrams. $1.25.

A reprint of a series of three lectures covering author's contributions to the kinetic theory of rarefied gases, delivered in 1933 and first published in 1934. Most of this material has been incorporated in textbooks on the kinetic theory of gases.

E. F. Lype, USA

1349. Grassmann, P., Counterflow condensation during simultaneous mass exchange in two-component mixtures (in German), *Ann. Phys.* (6) **7,** 1–2, 54–62, 1950.

Consideration is given to the concentration pattern in counterflow operation of a rectification column handling a two component mixture. The differential equations for the concentration in the liquid and vapor phase in terms of the reflux ratio are established. These are shown to be solvable in general in terms of the ordinary hypergeometric function under assumption that the vapor-liquid contraction equilibrium can be approximated linearly. Newman A. Hall, USA

1350. Dingle, R. B., The theory of the propagation of first and second sound in helium II. Energy theorems and irreversible processes, *Proc. phys. Soc. Sec. A,* **63,** part 6, 366A, 638–652, June 1950.

The conditions determining whether sound waves in helium II are propagated isothermally or adiabatically are investigated

An expression is found for the energy flux from a closed region. Effect of irreversible processes (due to viscosity and thermal conduction) on propagation of waves of first and second sound are investigated, both in an extended fluid and in a narrow tube.

From author's summary by Serge Gratch, USA

Heat and Mass Transfer

(See also Revs. 1232, 1342, 1349, 1409)

1351. **Wilson, L. H., and Miles, A. J., Application of the membrane analogy to the solution of heat-conduction problems,** *J. appl. Phys.* **21, 6, 532–535, June 1950.**

Paper describes method of solving two-dimensional steady-state heat-conduction problems by applying membrane analogy which is established by comparing appropriate differential equations. Construction and use of apparatus for measuring deflection of soap film and also method of constructing models, geometrically similar to the heat-conducting member, are described. Solution of three practical problems is discussed.

Method is claimed to be simpler and to require less expensive equipment than electrical network or geometrical-analog solutions and to have certain advantages over the numerical solution.

A. H. Žaludová, Czechoslovakia

1352. **Kruzhilin, G. N., Heat emission from a heating surface to a boiling monocomponent liquid with free convection** (in Russian), *Izvestiya Akad. Nauk SSSR Ser. tekh. Nauk*, no. 7, 967–980, July 1948.

1353. **Whalley, E., and Winter, E. R. S., The elementary theory of thermal diffusion,** *Trans. Faraday Soc.*, **46,** part 7, 517–526, July 1950.

The elementary treatment by Fürth [*Proc. roy. Soc. Lond. Ser. A*, 179, 1942] of transport phenomena by means of the mean free path method has been applied to mixtures of elastic sphere gases. Results are in good agreement with the exact theory except when the diameter and mass effects are in opposition. It is interesting to note that in case of mixture of isotopes, the elementary theory gives results which are in better agreement with experimental results than the exact theory.

Authors also apply method to case of multicomponent mixture. Since no detailed experimental results are available on thermal diffusion in such mixtures, the accuracy of the results cannot be assessed, but authors consider that their results could be used for an approximate analysis in such systems.

G. M. Lilley, England

1354. **McMahon, H. O., Thermal radiation from partially transparent reflecting bodies,** *J. opt. Soc. Amer.* **40, 6, 376–380,** June 1950.

Emissivity, transmissivity, and reflectivity are considered for heated, reflecting, partially transparent bodies. Kirchhoff's law, which in its usual form applies only to opaque bodies, is generalized to apply to matter of all kinds. An exact solution is found for the relationships between "apparent reflectivity" and true reflectivity and true transmissivity. Similarly, apparent transmissivity is expressed as a function of true transmissivity and true reflectivity.

R. M. Wingren, USA

1355. **Penner, S. S., The emission of radiation from diatomic gases. I. Approximate calculations,** *J. appl. Phys.* **21, 7, 685–695,** July 1950.

An approximate method for estimating radiant heat transfer from gaseous emitters is proposed. An average absorption coefficient is used for the effective width of an entire vibration-rotation band. The procedure for determining an average absorption coefficient in terms of the integrated absorption is justified, approximately for small optical densities and also for large total pressures where the spectral half width is no longer small compared with the rotational spacing.

R. M. Wingren, USA

1356. **Danforth, W. E., and Haddad, T. A., Radiation transfer considerations in the heating of a cathode sleeve,** *J. Franklin Inst.* **250,** 2, 135–145, Aug. 1950.

It is shown that a central heater must be at least at some 2300 C to heat a thoria cathode sleeve to 1800 C. This agrees with radiation theory. Temperature drop across the thin sleeve is negligible. Emissivity of the sleeve increases from about 0.26 when new and clean, to about 0.6 when darkened, as is the inside surface after heating for some time.

C. F. Bonilla, USA

1357. **Surinov, Yu. A., Integral equations of heat radiation and methods of calculation of radiant exchange in systems of "gray" bodies, separated by a diathermic medium** (in Russian), *Izvestiya Akad. Nauk SSSR Ser. tekh. Nauk*, no. 7, 981–1002, July 1948.

1358. **Hottel, H. C., Geometrical problems in radiant heat transfer,** *Nucl. Energy Power Aircr. Heat Transf. Lect.* II, 76–95, June 1949.

Paper illustrates method of solution when radiation and convection problems are mixed in a moderately rigorous fashion. Results show that for combined conduction, convection, and radiation the amount of heat loss depends on at least five dimensionless groups, sometimes seven. First part of paper reviews techniques of radiant-heat-transfer calculations. Last part gives solution to a problem which illustrates how the complexity of formulation grows when radiation and convection problems are mixed.

R. M. Wingren, USA

1359. **Bergelin, O. P., Brown, G. A., Hull, H. L., and Sullivan F. W., Heat transfer and fluid friction during viscous flow across banks of tubes—III. A study of tube spacing and tube size,** *Trans. Amer. Soc. mech. Engrs.* **72,** 6, 881–888, Aug. 1950.

Heat-transfer and pressure-drop data were obtained in the viscous flow region for seven tube banks. Tube banks used were: Three with $^3/_8$-in. diam tubes with a pitch-diam ratio of 1.25 with equilateral triangular, in-line square, and staggered square arrangements; two with $^3/_8$-in. diam tubes with a pitch-diam ratio of 1.5 with in-line square and equilateral triangular arrangements; and two with $^3/_4$-in. diam tubes with a pitch-diam ratio of 1.25 with in-line square and staggered square arrangements. Results show that, with an increased pitch with the same tube size, arrangement, and fluid velocity, the pressure drop and coefficient of heat transfer are lower. They show, further, that, with an increased tube diameter with same pitch-diam ratio, arrangement, and fluid velocity, the pressure drop and coefficient of heat transfer are also lower. A volumetric equivalent diameter in the Reynolds number gives better correlation of the heat-transfer data than does the tube diameter.

Byron E. Short, USA

1360. **Seban, R. A., Heat transfer to a fluid flowing turbulently between parallel walls with asymmetric wall temperatures,** *Trans. Amer. Soc. mech. Engrs.* **72,** 6, 789–795, Aug. 1950.

Starting from the solution for case of one adiabatic wall, given by Harrison and Menke, a solution for case in which heat transfer takes place through both walls has been obtained by a method of superposition of solutions. The former solution is extended giving

expressions for temperature distribution, and techniques are developed for determining heat transfer and temperature distribution in the more general case. Calculations in a range of Reynolds numbers from 10^4 to 10^6 and Prandtl numbers from 10 to 0.01, show that for low Prandtl numbers the magnitude of the heat-transfer coefficient differs considerably from that obtaining in the case of one adiabatic wall, but that the change is small for Prandtl numbers greater than unity. The extremes of variation occur when the walls are either at equal temperatures differing from the mean fluid temperature, or when the wall temperatures are equally above and below the mean fluid temperature. Sample calculations are shown for a specific example. Good agreement is shown between temperature distribution obtained experimentally by Corcoran and that predicted by present theory. The uncertainty concerning magnitude of the eddy diffusivities for heat and momentum near the center line of flow may lead to some discrepancies in the analysis. The predicted heat-transfer rates may be somewhat too high. Ione D. V. Faro, USA

1361. Berger, Jean, Thermal equilibrium of a porous plate cooled by injection of a cold fluid (in French), *C. R. Acad. Sci. Paris* **230,** 22, 1935–1937, May 1950.

Mathematical description of system has been treated in previous paper by Weinbaum and Wheeler [*J. appl. Phys.* **20,** 113–122, Jan. 1949; AMR 2, Rev. 522]. Author shows that maximum plate temperature decreases rapidly with increasing porosity, diameter of flow passages, or flow velocity of coolant gas. For a heat rate, normal to plate surface, whose distribution parallel to plate surface is a selected linear function, it is shown that longitudinal heat flow in the plate is not negligible.
F. E. Romie, USA

1362. Lutz, Otto, Graphical determination of wall temperatures for heat transfers through walls of arbitrary shape, *Nat. adv. Comm. Aero. tech. Memo.* 1280, 16 pp., Apr. 1950. (From *Z. Ver. dtsch. Ing.* **79,** 34, 1041–1044, Aug. 1935.)

Once the direction of heat flow at the wall boundaries is known, paper gives a simple graphical determination of the isotherms. These directions, however, must be found by a trial-and-error method which seems to require some skill. Three examples show application of the method. H. Wijker, Holland

1363. Manarini, Mario, On the heat equation (in Italian), *Boll. Un. mat. ital.* (3) **4,** 117–121, 1949.

Author points out that an integral representation given by E. Beltrami of class of solutions of the heat equation has properties similar to that of a volume potential. Then he gives two representations of solutions of the heat equation, first type having properties similar to those of a single-layer potential, and the second having properties analogous to those of a double-layer potential.
Courtesy of Mathematical Reviews F. G. Dressel, USA

1364. Bridgers, F. H., Combination of heat sources and sinks by the method of superposition, *Trans. Amer. Soc. mech. Engrs.* **72,** 3, 341–348, Apr. 1950.

Author presents a method to obtain the temperature pattern around a series of buried heat sources of equal temperature from the pattern around a single heat source, known by experiments. He derives $\theta_p = \Sigma\eta_i\theta_{ip}$, θ_{ip} being the temperature of a single source i at the point P, θ_p the real temperature at P, $\eta_i = \theta_i/\Sigma\theta_{ji}$ ($j = i$ included), θ_{ji} being the temperature of a single source j at the place of source i; so $\theta_{ii} = \theta_i$. Reviewer points out that this procedure generally does not give back the source temperatures from which one started ($\Sigma\eta_i\theta_{ij} \neq \theta_j$). It seems better to calculate the η_i's from a set of equations $\Sigma\eta_i\theta_{ij} = \theta_j$. Results of the method agree rather well with those of experiments given in paper. H. Wijker, Holland

1365. Hoge, Harold J., Compilation of thermal properties of wind-tunnel and jet-engine gases at the National Bureau of Standards, *Trans. Amer. Soc. mech. Engrs.* **72,** 6, 779–783, Aug. 1950.

A project for the preparation of a series of tables of thermal properties of gases is described. The scope of the project, tables now available, work in progress, and plans for the future are described. The lack of adequate data at high temperatures and also at high pressures is pointed out. From author's summary.

1366. Kochanovska, A., Investigation of thermal dilatation of cubic metals, *Physica* **15,** 191–196, Apr. 1949.

For measuring thermal dilatation of aluminum and iron in different directions, x-ray back reflection photographs were made of different lines in the temperature range of 22 to 366 C. In order to obtain these lines at high Bragg angles as much as possible, different wave lengths were used. These were in the case of aluminum Cu-, Co- and CrKα1 for the lines (115), (024), and (222) respectively, and in case of iron Co- and CrKα1 for the lines (013) and (112) respectively. The film holder was arranged in such a way that four exposures could be made on each film, the first being a reference pattern of the lines of the investigated metal obtained at room temperature. A small furnace was used for heating, mounted in a metallic container with a window of Lindemann glass for passing the x-rays and being evacuated to prevent errors by oxidation and air flow. Results are that, for aluminum, thermal dilatation is same in all directions investigated, but for iron there are differences of about 5% out of the possible experimental errors. This indicates that iron is not precisely cubic at all temperatures, and this might be the case for other cubic metals. Albert Kochendörfer, Germany

1367. Hirst, W., and Kerridge, M., A calorimeter for the measurement of small amounts of heat liberated slowly, *J. sci. Instrum.* **27,** 6, 161–163, June 1950.

This calorimeter was devised for measurement of heat quantities of the order of 1 to 10 cal. Special feature claimed is that heat leakage remains approximately constant for several hours. It is therefore possible from the cooling curves to distinguish between processes such as heat-of-wetting and heat-of-chemical reaction. The calorimeter consists of three metallic concentric cylinders A (innermost), B (intermediate), C (outermost immersed in water bath). The material to be examined is sealed in a glass capsule and fixed in A by means of a holder and wetting liquid added. A lid through which projects an impact rod and calibration heater, prevents escape of vapor. A is suspended from lid on B by a thermally insulating Fernico-glass-Fernico seal. B is wound with resistance wire. Space between A and B is evacuated through B lid. Insulated platinum resistance thermometers are fastened to the side of A and at bottom of B. This makes it possible to control and measure temperatures of B with respect to A. B is suspended from lid on C. Space between not evacuated. Impact rod is continued through the three lids. Dimensions of A, height about 8 cm, diam 1; of C, 15 and 30 cm. Platinum resistances on A and B form two arms of a Wheatstone-bridge circuit, output of which is connected to an amplifier involving the use of a galvanometer-photocell set up with a single thyratron valve. The circuits are so arranged that, when the temperature of A rises, the output of amplifier is passed through heating element on B, causing its temperature to increase until

initial temperature difference between A and B is restored. Under steady conditions a small current through coil keeps temperature of B higher than that of C. Hence temperature difference between A and B becomes practically independent of the absolute temperature of A. Performance was tested with and without amplifier. Heat leakage of one test was 1.29 cal/hr over a 40-min period. 7.2 cal were then added to C by means of test coil for next 40 min. Heat leakage for third period of 40 min was 1.18 cal/hr. Heat leakage in 2 hr changed only 0.11 cal/hr. Curves showing the heat of wetting of titania powder by stearic/benzene solution and of cuprous oxide with same solution show distinct differences as to nature of surface interaction. Such differences are not revealed by calorimeters similar to those of Harkins and Dahlstrom. Errors are those common to most calorimeters. Wm. F. Seyer, USA

Acoustics

(See also Revs. 1029, 1350, 1380)

1368. **Lottermoser, Werner, Electroacoustical measurements on Barockorgans of Oberschwaben: I. Sound pressure diagrams, II. Analytical sound investigations** (in German), *Z. Naturforsch.* **3a, 5a,** nos. 5, 3, 298–308, 159–168, May 1948, Mar. 1950.

I. Sound pressure in the middle of the church was measured. Relatively small wind pressure causes intensities up to 1.8 dynes/cm². Writer recommends imitating characteristics shown for modern organs. II. Overtones of the different pipes were measured. Value of these famous organs lies in the predominance of the overtones in the broad region in the neighborhood of 1000 cps. Many diagrams give valuable information in both papers.
R. Vermeulen, Holland

1369. **Muncey, R. W., The use of three-dimensional models in room acoustics,** *J. acous. Soc. Amer.* **22,** no. 4, 510–511, July 1950.

Author shows that the acoustical response of a room or other closed space will be simply related to that of a model or replica of room whose dimensions are scaled down by a factor $1/m$ if the model is investigated at a frequency m times the frequency for full-scale room. This will hold true if corresponding areas in the model and room are lined with materials having equal acoustic impedances at the frequency $m\nu$ (for model) and ν (for room). To reviewer, this conclusion appears intuitively obvious. Main difficulty in using models would appear to be in obtaining lining materials which at model frequency, presumably ultrasonic, have not only same normal impedance but also same dependence with angle of incidence as materials used in full-scale room at audiofrequencies. An obvious difficulty might be the occurrence of flexural waves in a material at ultrasonic frequencies which, however, might not occur in the audiorange. Author points out methods of allowing for differences in air absorption and sound velocity in such model experiments. A. London, USA

1370. **Bolt, R. H., Doak, P. E., and Westervelt, P. J., Pulse statistics analysis of room acoustics,** *J. acous. Soc. Amer.* **22,** 3, 328–340, May 1950.

Theory of statistics of response of rectangular room with rather hard walls, using image source method, gives expected number of pulses received as function of time, mean free path, reverberation time, and "period of resolution" of separated impulses. Experiments with 3600 cps and 2 m sec pulse duration are compared with calculations.

Analysis of room acoustics using pulsed tones has been done before, but reviewer believes the theory given opens new ways.
R. Vermeulen, Holland

1371. **Mendousse, J. S., On the theory of acoustic radiation pressure,** *Proc. Amer. Acad. Arts Sci.* **78,** 3, 148–164, Apr. 1950.

Acoustic radiation pressure is usually computed by approximation methods, the accuracy of which depends on the "amplitude" of motion. This theory begins wtih a rigorous definition of the amplitude of the whole motion. First- and second-order terms in the series expansion of necessary quantities with respect to amplitude are rigorously derived. Theory development brings out a mass energy relation, defining the equilibrium of a volume V of fluid by the average position of its boundaries (with respect to time), the difference of the potential energies inside the volume V is equal to $^1/_2\,(mc^2)$ where c is the wave velocity and m the time average increase of mass caused by the motion inside of V. A formula is obtained for radiation pressure on the walls of a container which participates in the oscillation of the fluid.
From author's summary by C. O. Dohrenwend, USA

1372. **Bogert, B. P., Classical viscosity in tubes and cavities of large dimensions,** *J. acous. Soc. Amer.* **22,** 4, 432–437, July 1950.

A method is derived for determining the viscous losses in acoustic wave guides for both plane wave transmission and higher-order modes. The (00), (01) mode in a round tube, (10) in a rectangular tube, are considered in addition to losses in cavities. Technique first assumes the approximate particle velocity distribution which would exist in absence of any losses. Next, rate of energy dissipation as a function of distance along guide is computed and equated to space rate of decrease of energy transmitted through guide. The Rayleigh dissipation function is used to evaluate the latter, while former is evaluated from applying Rayleigh's treatment of viscous losses at a rigid wall to the approximate particle velocity distribution, in such a way as to give zero tangential velocity at the wall. First-order solutions are then obtained for the required attenuation coefficients.
A. London, USA

1373. **Takesada, Yoshihito, On the absorption of high power sound waves,** *J. acous. Soc. Amer.* **22,** 4, 515–516, July 1950.

Author attributes excessive absorption of high power sound in water to reflection from nonresonant air bubbles therein. He calculates reflection and attenuation coefficients in terms of changes in acoustic impedance of the composite medium, produced by changes in bubble content along the beam. He finds that for bubbles of given size, attenuation is greater for fewer bubbles per unit volume. Reviewer believes attenuation coefficient is introduced incorrectly, making results fallacious.
A. O. Williams, Jr., USA

1374. **McSkimin, H. J., Ultrasonic measurement techniques applicable to small solid specimens,** *J. acous. Soc. Amer.* **22,** 4, 413–418, July 1950.

Description of method of measuring ultrasonic velocities in small samples of solids, based on determination of phase shift accompanying internal reflection of ultrasonic wave from ends of sample. For two consecutive reflections at frequency such that the incident wave and first echo are received in phase, velocity is given by $V_0 \equiv 2Lf_0/[n_0 - (\varphi_0/\pi)]$, where n_0 is integer, f_0 is frequency, φ_0 is phase change on reflection, L is sample length. Method described is used to determine f_0 and φ_0. Specimen is held by quarter wave-length seals between two buffer rods, at free ends of which are attached driving and receiving crystals. Use of quarter wave-length seals minimizes φ_0. In-phase frequency, f_0, is

determined by use of variable mercury delay line in parallel with specimen between oscillator and detector. Delay line enables finding frequency at which incident pulse and first echo are in phase to within one degree. Phase shift, negligibly small as first approximation, is measured accurately by use of delay line and phase shifter. Initial pulse balance is destroyed by removal of one buffer and seal, and phase change by shifter required to restore balance is direct measure of φ_0. Crystal at end of remaining buffer is used both as driver and receiver. Method is practicable for measuring temperature coefficients of velocity, and velocity changes due to external causes (e.g., applied magnetic field, etc.). Method is one of high accuracy. P. Tamarkin, USA

1375. Daniels, Fred B., On the propagation of sound waves in a cylindrical conduit, *J. acoust. Soc. Amer.* **22, 5, 563–564, Sept. 1950.**

Author, in a previous paper [title source, **19**, p. 569, 1947] found the acoustical impedance for three shapes, a sphere, cylinder, and a narrow rectangular box. Mawardi [op. cit., **21**, p. 482, 1949], using some of author's previous results, obtained, via an approximate technique, the characteristic impedance of a tube filled with wires. Present paper analyzes the case of a cylindrical conduit filled with air at 20 C.

The conduit is represented by an electrical transmission line. Effects of viscosity and heat conduction are introduced separately by varying series and parallel elements of the equivalent T section. Numerical values of characteristic impedance and propagation constant are computed as a function of the product of frequency raised to one-half power multiplied by radius of conduit. Results obtained are presented in five graphs.

K. M. Siegel, USA

1376. Hubbard, Harvey H., Sound measurements for five shrouded propellers at static conditions, *Nat. adv. Comm. Aero. tech. Note* **2024, 31 pp., Apr. 1950.**

In an article, "Silencing of propellers by thrust relief" [*Aero Digest*, **33**, 4, Oct. 1938], Max Munk indicates that a propeller operated inside a short duct or shroud will produce greater thrust and less noise. Sound measurements have been made on shrouded propellers, which show that when the flow through the shroud is good, the sound pressures are reduced to one half (6 decibels). However, it is difficult to maintain good flow through the shroud, and when the flow separates at the lip of the shroud, the sound is greater than for an unshrouded propeller. A. Regier, USA

1377. Mokhtar, M., and Messih, G. A., The acoustic characteristics of conical pipes, *Proc. phys. Soc. Lond. Sect. B*, **62,** part 12, 360B 793–799, Dec. 1949.

Paper describes an experimental investigation on slightly tapering conical pipes used as sound transmitters, undertaken in order to determine their acoustic characteristics, their end corrections, and the pressure variation along the inside of the pipes.

A hot-wire anemometer and a Pitot pressure tube were used for recording particle velocity and pressure variation in the acoustic field. The results obtained for the resonant frequencies show fair agreement with the theoretical formula $kr = -\tan kl$, where r is the radius of the throat of the conical pipe, l is its slant length and $k = w/c$, w being the angular frequency and c the velocity of sound.

The recording of the acoustic pressure along the axis of the conical pipes shows that the nodes inside are places of minimum but not zero pressure, and that they are not equidistant. End corrections are deduced from these pressure curves. Correction is always positive, being less than that of a cylindrical pipe of diameter equal to that of mouth of the cone. It increases with angle of the cone and with order of the overtone to which the conical pipe resonates. From authors' summary

Ballistics, Detonics (Explosions)

(See also Rev. 1248)

1378. Garnier, Maurice, Long range ballistics (in French), *Mémor. Artill. fr.* **24, 1, 159–198, 1950.**

This authority on exterior ballistics recalls the many additional considerations needed when one extends the traditional methods for moderate ranges to include effects conspicuous only for long ranges, such as curvature of the earth with divergence of verticals, rotation of the earth, variation of gravity with altitude, etc. He proposes using, in place of rectangular coordinates, the geodesic abscissa and geodetic altitude. The necessary formulas for converting to new coordinates are here developed. In practical cases the measuring of distances on the earth by telemetric methods and observation of azimuths by direct sights introduce the need for special adjustments. A further important correction is called for in any method which interprets terrestrial positions upon a flat map. Author, as is his custom, illustrates all details by concrete numerical examples. Albert A. Bennett, USA

1379. Barton, M. V., The effect of variation of mass on the dynamic stability of jet-propelled missiles, *J. aero. Sci.* **17, 4, 197–203, Apr. 1950.**

The equations of motion in a vertical plane for a missile having a constant rate of loss of mass are developed and solved for the simplified case of a symmetrical rocket missile. It is shown that the disturbance of the variable mass missile damps out more rapidly than it does for the constant mass missile, indicating that former is more stable.

From author's summary by A. Petroff, USA

1380. Scorer, R. S., The dispersion of a pressure pulse in the atmosphere, *Proc. roy. Soc. Lond. Ser. A*, **201,** 1064, 137–157, Mar. 1950.

An explosion on the ground is represented mathematically by a Fourier integral, corresponding to introduction of a large volume into the atmosphere at that point. Resulting pulse is calculated for various distances for a model atmosphere consisting of a troposphere with a constant lapse-rate of temperature and an isothermal stratosphere. It is composed of those oscillations that can be propagated horizontally as gravity waves in this model atmosphere, namely, those of period exceeding a cut-off period of 111 sec. Higher frequencies escape through stratosphere, but owing to dispersion, the recorded pulse at a point on the ground consists of a series of waves of decreasing amplitude and period, terminating with a period of 12.7 sec.

Results are compared with oscillations observed on occasion of the fall of the Great Siberian Meteorite, and the energy which it is estimated to have communicated to the atmosphere is about 4×10^{24} ergs, only a fraction of which resided in gravity wave.

From author's summary by H. H. M. Pike, England

1381. Taylor, G. I., The formation of a blast wave by a very intense explosion, I—Theoretical discussion, II—The atomic explosion of 1945, *Proc. roy. Soc. Lond. Ser. A*, **201,** 1065, 159–186, Mar. 1950.

A general study of spherical blast waves is tedious because of entropy variations. For very strong shocks, the front boundary conditions are very simple and, if starting conditions can be ignored, a similarity solution is possible. For a wave of constant total energy, initially produced in a small volume, all dependent varia-

ties equal products of pairs of functions of the form $f(xt^{-2/5}) \cdot g(t)$. The functions f involve only one parameter γ (ratio of specific heats) and are calculated for $\gamma = 1.4$. At any instant the fluid velocity increases almost in proportion to radius at all points. Coming in from the front, pressure diminishes rapidly to about half its peak value and then remains nearly constant, while density continues to diminish and temperature rises without limit near the center.

The functions g give values at the front and depend very simply on t; e.g., for $\gamma = 1.4$ the shock pressure $p = 0.155\ E/R^3$, where R is radius and E total energy of the wave. The method is invalid for $p < 10$ atm and does not fit results for orthodox explosives for $p > 100$ atm because of initial conditions. In between, experimental pressures vary with R in the right manner but are about double those predicted because energy dissipated as heat in the early stages is much less than that calculated. This energy dissipation $E_1 + E_2$ $(=\gamma E_1)$ is calculated to reach $0.45E$ when $p = 20$ atm.

In part II the theory is applied to the atomic explosion in New Mexico and photographs are shown. Prediction that in the early stages $R \sim t^{2/5}$ is confirmed with remarkable precision and a good estimate of E is obtained using $\gamma = 1.4$. For such high pressures the true value of γ is < 1.3 and the discrepancy may be due to energy transfer by radiation. The density distribution in the ball of fire after return to 1-atm pressure is calculated and leads to correct rate of buoyant rise of ball. H. H. M. Pike, England

1382. **O'Day, Marcus, Instrumentation of V-2 rockets to obtain data on the physics of the upper air,** *Proc. seventh int. Congr. appl. Mech.* **2**, part 2, 515–524, 1948.

Soil Mechanics, Seepage

(See also Rev. 1223)

1383. **Childs, E. C., and Collis-George, N., The permeability of porous materials,** *Proc. roy. Soc. Lond. Ser. A*, **201**, 1066, 392–405, Apr. 1950.

Attempt to express the permeability of a porous material as a function of the pore-size instead of the particle-size distribution, as generally done in the past (e.g., Kozeny's formula). Pore-size distribution can be obtained from the experimental curve of moisture content plotted against pressure deficiency due to surface tension on air-water interfaces in equilibrium in the pore spaces. Thence, through statistical considerations, a formula can be established for the permeability; a direct measurement is necessary to determine a numerical coefficient. Some experiences conducted by authors should confirm validity of formula, whose value is perhaps merely theoretical. Duilio Citrini, Italy

1384. **Wyllie, M. R. J., and Rose, Walter D., Some theoretical considerations related to the quantitative evaluation of the physical characteristics of reservoir rock from electrical log data,** *J. petrol. Tech.* **2**, 105–118, Apr. 1950.

Authors give a theoretical basis to quantitative interpretation of electrical log data, expressed in formation resistivity factor and cementation factor, which are shown to be related to rock textural parameters in expressions of capillary phenomena.

On assumption that tortuosity applicable to fluid flow of wetting phase in a porous medium is same as the tortuosity affecting electrical conductivity through the fluid in the same medium, they derive an expression for formation factor F of the form $F = T^{1/2}/\varphi$ (where T is tortuosity and φ porosity of the medium), from which is shown that the common relationship $F = \varphi^{-m}$ is of no fundamental significance.

Applying same assumption, they further derive an expression for the resistivity index I in terms of saturation and ratio of the wetting liquid tortuosities at 100% saturation and the saturation S_w considered: $I = (T_e/T)^{1/2}S_w$. Further, a relationship for the magnitude of a formation permeability K is proposed: $K =$ constant $(1/P_c^2 F^{(2-2/m)} S_w)$ (where P_c is capillary pressure, m cementation factor exponent). In a paragraph about development of practical applications, authors describe the estimation of connate water saturation, of porosity, and of permeability from electrical log data. F. C. de Nie, Holland

1385. **Teng, C. Y., Determination of the contact pressure against a large raft foundation,** *Géotechnique Lond.* **1**, 222–228, Dec. 1949.

Many theories have been stated for determination of the contact pressure against the base of raft foundations, most of them based on assumption that pressure is proportional to settlement of the foundation. Experimental verification for such raft foundations have not yet been published.

Present paper gives for the first time the result of direct observations drawn from the raft foundation of the Nonoalco Power Plant in Mexico City. The subsoil of this town is well known to geotechnical engineers: it is a highly compressible volcanic silty clay which causes extensive settlements. The raft foundation consists of a rectangular gridwork of intersecting steel trusses resting on a thin slab of reinforced concrete. This construction can be considered as a flexible elastic system which allows an easy determination of deformations and of stresses developed in the trusses. Consequently, sufficient precision could be obtained for determination of the forces necessary to deform the raft and for evaluation of contact pressures.

Observations which began in 1931 were closed in 1944, and diagrams were made for differential settlements and for contact pressures. Although the dead and live loads were practically constant, contact pressures changed continually during this period. Author concludes that contact pressure depends largely on unpredictable variations in compressibility of subsoil, and that a reasonable evaluation is more important than theoretical and mathematical speculation about soil reaction.

Aurel A. Beleş, Rumania

1386. **McNown, John S., and Hsu, En-Yun, Effect of partial cutoff on seepage rates,** *Trans. Amer. geophys. Un.* **31**, 3, 468–472, June 1950.

Author determines the effectiveness of a partial impervious barrier in a seepage channel. Using conformal transformation (Schwarz-Christofel theorem), variation of the quantity of flow with extent of penetration and geometrical proportions of channel are obtained in terms of elliptic functions; analytical and graphical results are given. Only for effectively short channels do cutoff walls reduce rate of flow significantly.

Reviewer believes the same problem was solved by H. Rossbach, also using conformal transformations ["Ueber die unter einem Damm durch eine horizontale Parallelschicht sickernde Wassermenge und die Auftriebsdruckverteilung an der Dammbasis," *Z. angew. Math. Mech.*, p. 65, Apr. 1942].

L. J. Tison, Belgium

1387. **Biésel, F., Equations of relatively rapid flow through permeable matter** (in French), *Houille blanche* **5**, 2, 157–160, Mar.-Apr. 1950.

In place of the connection of body force and velocity known in hydrodynamics, the body force: grad $U - \lambda$ grad φ is assumed, where φ is velocity potential. Then the Bernoulli equation has an additional term $+\lambda\varphi$. The determination of φ gives, then, a rela-

tion for wave length and coefficient λ of permeability, frequency and damping of wave being known.

H. Schlechtweg, Germany

1388. Vibert, A., Water motion in soil. New considerations on the discharge of a free underground water level flowing into a well or a drain (in French), *Génie civ.* **76**, 9, 160–165, May 1949.

1389. Polubarinova-Kochina, P. Ya., On unsteady filtration with surfaces of separation (in Russian), *Dokladi Akad. Nauk SSSR* **66**, 2, 173–176, May 1949.

1390. Croney, D., and Coleman, J. D., Soil thermodynamics applied to the movement of moisture in road foundations, *Proc. seventh int. Congr. appl. Mech.* **3**, 163–177, 1948.

1391. Deacon, E. L., The measurement and recording of the heat flux into the soil, *Quart. J. roy. met. Soc.* **76**, 330, 479–483, Oct. 1950.

Relatively simple instruments for measuring heat flux into or out of soil are described together with methods of calibration. One form consists of a suitable metal disk of sandwich construction producing a thermal e.m.f. proportional to heat flux through disk, which is placed in soil just below surface. Details are also given of another form of flux plate which is suitable for continuous recording using a thread recording galvanometer.

From author's summary

1392. Meinzer, Oscar E., and Wenzel, Leland K., Movement of ground water and its relation to head, permeability, and storage, *Hydrology*, ed. by O. E. Meinzer, New York, Dover Publications, 444–477, 1949.

A reprint of article "Hydrology," by authors in *Physics of the Earth*, **IX**, published in 1942. Article is a very useful summary of subject covered by title. Various formulas relating permeability and discharge from a well are discussed, together with such matters as extent of cone of depression and behavior of ground water in vicinity of wells. For a more detailed treatment, reference should be made to excellent monograph by junior author, published in 1942, as *U. S. Geol. Survey Water Supply Paper* 887.

A. W. Skempton, England

1393. Hvorslev, M. Juul, Subsurface exploration and sampling of soils for civil engineering purposes, *Ways Exp. Sta. Rep.*, xi + 521 pp., Nov. 1949.

Report represents an up-to-date complete coverage of methods and apparatus used in soil investigation for foundation engineering. Material is presented under two main divisions. The first deals with methods and procedures of subsurface exploration and sampling; the second gives a detailed description of the sampling apparatus in common use. Second part also includes methods and apparatus used in ocean-bottom sampling and exploration for oil and minerals, as well as a discussion of the different soil-sampling methods. Emphasis is placed on dependence of type of soil on particular method to be used. An extensive bibliography is also included.

Eben Vey, USA

1394. Croney, D., Lewis, W. A., and Coleman, J. D., Calculation of the moisture distribution beneath structures, *Civil Engng. London* **45**, 524, 103–106, Feb. 1950.

Paper describes two approaches to problem of calculating the equilibrium distribution of moisture in saturated soils (these include most undisturbed clays). Methods are based, respectively, on soil moisture suction and on the consolidation characteristics of the soil, both of which can be determined by laboratory tests. The work was undertaken in connection with soil drainage and problem of estimating the thickness of roads and airfield runways.

From authors' summary

1395. Krynine, D. P., Vertical and horizontal shearing stresses in earth masses, *Proc. sec. int. Conf. Soil Mech. Found. Engng.* **7**, 21–24, June 1948.

In a homogeneous, undisturbed, semi-infinite soil mass, horizontal and vertical planes are principal planes subjected to normal stresses which are proportional to the depth below the free surface. A "sudden" excavation disturbs these conditions in a region adjacent to the excavation. Paper develops qualitative considerations concerning effects of such disturbances. Author developed the same ideas further in a later paper [see AMR 3, Rev. 1594].

O. Hoffman, USA

1396. U. S. Army, Corps of Engineers, Soil compaction investigation, *Ways Exp. Sta. tech. memo.* **3-271**, Rep. no. 1, 43 pp., no. 2, 49 pp., no. 3, 22 pp., no. 4, 22 pp., Apr., July, Oct. 1949; Feb. 1950.

These four papers contain a comprehensive investigation on the compaction and mechanical characteristics of: (*a*) a clayey sand, (*b*) a lean silty clay, (*c*) a clean sand subgrade, and (*d*) two natural subgrades and two constructed subgrades of cohesive soils. The test fills have been made by sheep's-foot rollers and rubber-tired rollers of various pressure intensities. During construction, moisture determinations, density tests, and field-in-place C.B.R. tests were made for each case. The same tests were likewise made in the laboratory; and, eventually, unconfined and triaxial compression tests were made on undisturbed samples compacted in the field and on material compacted in the laboratory.

Reports give very complete information on comparison of results of all these tests, and state substantial conclusions regarding use of the various types of field compaction equipment.

R. Spronck, Belgium

1397. Doll, H. G., The microlog—a new electrical logging method for detailed determination of permeable beds, *J. petrol. Tech.* **21**, 6, 155–164, June 1950.

1398. de Mello, V. F. B., and Lambe, T. William, The procedure and significance of chemical and mineralogical tests on soils, *Proc. Amer. Soc. Test. Mat.* **49**, 977–992, 1949.

Paper presents modifications of procedures for chemical and mineralogical soil tests for use of soil mechanicians. Tests covered are identification of soil minerals, determination of hydrogen-ion concentration, detection of deleterious substances, and determination of exchange capacity and exchangeable bases. An extensive bibliography is included.

C. M. Duke, USA

1399. Krynine, D. P., Some capillary phenomena in sandy materials, *Highway Res. Board, Proc. 29th ann. Meeting*, 520–530, 1950.

The problem of changing moisture conditions in subgrades of highways and airport runways is important in their design. Present information regarding the part which capillary movement plays in such moisture changes is incomplete. In order to supply needed basic information, small scale tests were made to study both horizontal and vertical capillary movement of water through tubes containing three types of sand. Effects of boundary conditions and water vapor on moisture content were observed. Important conclusions, limited to conditions of tests, indicate (1) difference between vertical and horizontal capillarity consisted mostly in speed of movement; (2) boundary effect in vertical tubes

when outflow face was reached resulted in temporary increase in total moisture, followed by decrease to equilibrium point.

Frederick J. Converse, USA

Micromeritics

(See also Rev. 1383)

1400. **Bianchi, B., The granulometer separator** (in Italian), *Metal. Ital.* **42**, 4, 128–132, Apr. 1950.

A device is described for giving a visual estimate of particle size distribution in a water suspension. A suspension held in a pipette is transferred to a number of fixed tubes of small diameter at stated intervals of time determined from Stokes' law or from Newton's law (if the particles are large and their motion turbulent). The transfer pipette is fixed to a rack and pinion arrangement and can be moved to the mouths of the smaller fixed tubes arranged in a line. The volume of particles in each of the latter traces a size-frequency distribution. Author presents design data and experimental results, together with a table of correction factors when the motion of the particles is turbulent. While the method has no advantages over the usual hydrometer technique used in this country, the method could be useful for routine analysis by untrained personnel. J. M. DallaValle, USA

Geophysics, Meteorology, Oceanography

(See also Revs. 1231, 1382)

1401. **Pekeris, C. L., Free oscillations of an atmosphere in which temperature increases linearly with height,** *Nat. adv. Comm. Aero. tech. Note* 2209, 24 pp., Oct. 1950.

Note concerns a paradox which arises in the theory of long period atmospheric oscillations when it is assumed that temperature of the atmosphere increases linearly with height without limit. All solutions for the equation for the hydrodynamical divergence then correspond to waves in which the energy per unit column is infinite, although there are grounds for expecting that one solution in which it is finite will exist. This paradox occurs because a term arising from vertical acceleration is ordinarily neglected in the equation; if temperature increases with height, this term becomes important at sufficiently high levels. Some calculations in which this term is taken into account are described and applications to the solar semidiurnal oscillation made. It seems to reviewer that at high levels where term depending on vertical acceleration is important, other terms depending on viscosity and similar absorbing agencies will also be of importance and that the effect of these terms may be to modify the motion considerably.

M. V. Wilkes, England

1402. **Kuo, Hsiao-Lan, The motion of atmospheric vortices and the general circulation,** *J. Meteor.* **7**, 4, 247–258, Aug. 1950.

Paper deals with the motion of vortexes in upper troposphere as deduced from derived equations of motion where certain assumptions are made regarding vortex structure and motion of the bounding fluid.

Assuming vortex to have a solid boundary, author derives the general Bernoulli equation for pressure on the boundary and shows that solution depends upon a determination of the stream function, which in turn can only be obtained when the absolute vorticity is uniformly distributed throughout the basic current. Two situations are then considered in which absolute vorticity is constant. In the first, the Coriolis parameter and relative vorticity are constant (sum of these components is the absolute vorticity), whereas in the second, these quantities are linear functions of the longitudinal component y. For these situations, the stream functions are evaluated and the equations of motion for the vortexes obtained.

Author next considers the more realistic case in which absolute vorticity is not uniform in the basic current, and vortex has no solid boundary. No exact solution to the vorticity equation could be found, but direction and magnitude of the forces on vortexes arising from the vorticity distribution were obtainable. It is shown that cyclonic vortexes tend to move northward to regions of higher vorticity, whereas anticyclonic vortexes move southward to regions of lower absolute vorticity. Occasionally strong westerly jets in upper troposphere give rise to definite meridional zones of minimum and maximum absolute vorticity. Under such conditions, cyclonic vortexes might be driven southward and anticyclonic vortexes northward, and the appearance of closed cold and warm cells is then observed in upper troposphere.

Warren W. Berning, USA

1403. **Riehl, Herbert, A model of hurricane formation,** *J. appl. Phys.* **21**, 9, 917–925, Sept. 1950.

Author has made use of the increased quantity of meteorological data in tropical regions to describe more thoroughly than heretofore the fields of pressure, temperature, and velocity associated with tropical storms. By summarizing the results of many analyses of meteorological charts, he shows that the tropical storm represents a large scale vertical circulation through which a large portion of the air over a tropical ocean may pass in the course of a few days. He uses chart analysis to explain processes which produce the vertical circulation and to specify conditions required for development of a tropical storm. Resulting empirical generalizations, reviewer believes, represent a significant advance in understanding the process of tropical cyclogenesis. However, author also employs largely intuitive physical reasoning which treats the equation of horizontal motion but omits from discussion the other hydrodynamic equations. Reviewer feels this procedure is justified only in cases in which individual terms in the equation can be specified independently of the others; however, in the tropical storm such independence of terms is doubtful.

Robert G. Fleagle, USA

1404. **Fleagle, Robert G., A theory of air drainage,** *J. Meteor.* **72**, 6, 227–232, June 1950.

A theoretical treatment of down-slope winds is given which includes adiabatic compression and radiational cooling. Results indicate that the maximum drainage air speed is proportional to cotangent of elevation angle of the slope as opposed to cosine of the angle (according to Prandtl), or directly to slope of the ground (according to Jeffreys). A dynamic solution which includes the inertia term, shows wind speed to be a damped periodic function of time when friction depends upon first power of wind speed. Author believes that sketchy evidence supports this periodicity. An equilibrium solution using second power of wind speed for friction eliminates periodicity and appears to be more realistic. Derivation is based on particle mechanics so that predicted air speeds refer to successive motions of the same parcel rather than providing the changes in speed at a fixed place.

L. Machta, USA

1405. **Woeber, Walter A., The wind speed in structural calculations** (in German), *Öst. Ing.-Arch.* **95**, 7/8, 53–59, Apr. 1950.

The correlation between wind-tunnel model tests and atmospheric conditions, as affected by Reynolds number (scale effect) and Richardson's number (thermal gradient effect) is discussed. Trying to use drag coefficients found in wind tunnels, author proposes to determine the wind speed of storms from a statistical

point of view. From a large number of measurements, he concludes that the wind structure is nonperiodic. A maximum root mean square wind speed and a gust impulse factor are defined and used in the force expressions to represent the dynamic pressure, although the justification for the use of the gust impulse factor is not fully given. A review of the literature of the variation of wind speed with height above ground is attached.

Y. C. Fung, USA

1406. **Press, Frank, Ewing, Maurice, and Tolstoy, Ivan, The Airy phase of shallow-focus submarine earthquakes,** *Bull. seism. Soc. Amer.* **40, 2,** 111–148, Apr. 1950.

An extension of theoretical work of Lamb and Pekeris shows that a prominent phase (Airy phase) consisting of a group of waves of period 9–11 seconds, traveling across the ocean with a velocity of approximately 0.7 of speed of sound in water, should be present on seismograms of shallow-focus submarine earthquakes. This arrival corresponds to normal mode propagation at a stationary value of group velocity through the acoustic system consisting of ocean and rigid ocean bottom.

A phase fitting this description has been observed on the Milne-Shaw instruments at Bermuda for a series of Dominican Republic shocks. The Wenner seismographs at Huancayo recorded the Airy phase of a shock southwest of the Galapagos Islands.

The study of the Airy phase of a sufficient number of seaquakes would provide information concerning the nature of the ocean bottom, since the latter affects both the periods and velocities of the Airy phase.

From authors' summary by Walter H. Munk, USA

✎1407. **Bartels, Julius, Geophysics (Geophysica), I, II** (in German), Naturf. Mediz. Deutschl. 1939–1946, vols. 17, 18; Wiesbaden, Dieterich'sche Verlagsbuchh., 237 pp., 307 pp., 1948.

The two volumes contain a description of development of geophysical research in Germany from 1939–1946 with detailed bibliography. Most sections contain data on instruments. First begins with the form and figure of the earth, followed by results on gravity and terrestrial magnetism. A section on the ionosphere contains applications to radio-wave transmission and on forecasts of ionospheric disturbances. First volume concludes with applied geophysical methods and gives some examples of finding of structures of economic importance in Germany. In second volume, a section on seismology includes developments of theory of elastic waves, their propagation in the earth, and effects of absorption and damping. Laboratory experiments show, e.g., propagation of elastic waves across a discontinuity and the wave fronts involved in this case. Records produced by artificial explosions are discussed. A special section deals with investigations of ground movements produced by earthquakes, machinery, traffic and artificial explosions and effects on buildings, and includes vibrations of chimneys and bridges from wind. One section covers vibration meters, accelerographs, geophones and related instruments. Propagation of sound waves in atmosphere, including dispersion, is discussed. A section on oceanography contains information on instruments for measurements in open ocean as well as at the coast, data on tides and sea level measurements. A section on hydrology gives data on rainfall and on the balance of water between continents, rivers, and oceans. Last two sections discuss geodetic methods and ozone in atmosphere.

B. Gutenberg, USA

1408. **Vening Meinesz, F. A., Earth's crust deformations in geosynclines,** *Proc. kon. Ned. Akad. Wet.* **53,** 1, 27–46, 1950.

Author compares his own 1930 hypothesis that geosynclines are initiated by buckling of the earth's crust with Bijlaard's 1935 hypothesis. According to Bijlaard, local plastic deformations occur first, analogous to flow lines in steel plates. After thickening of a locally plastically deformed strip, isostatic balance causes eccentricity of compressive force. This causes crust to bend downward and to form a geosyncline. Assuming viscous deformation of crust, author calculates vertical movement of crust in geosyncline as function of time. He also examines possibility of second crustal wave adjacent to primary geosyncline, as occurs in oil geosynclines of Java and Sumatra. He concludes that Bijlaard's hypothesis is more likely and can explain many questions otherwise difficult to account for. Throughout whole phenomenon it gives a satisfactory explanation of downbulging of crust, which belts of negative gravity anomalies, discovered by author, lead him to suppose. It also explains intricate pattern of these belts in Indonesia.

Reviewer agrees, of course, with these qualitative statements, but he should like to see calculation improved quantitatively, among other things, by taking account of increasing counterpressure of substratum during downward bending of crust.

P. P. Bijlaard, USA

1409. **Trombe, F., Foex, M., and La Blanchetais, Ch. H., Solar energy furnaces** (in French), *C. R. Acad. Sci. Paris,* **231,** 1, 44–46, July 3, 1950.

Authors describe furnaces heated by solar radiation. In three previous papers, mentioned in present paper, they have shown various effects of solar radiation, in particular, heating of an insulated cavity by solar energy. In this paper they give results of experiments carried out at Mont-Louis (France).

Procedure of heating furnaces is by means of a parabolic mirror with a horizontal axis and having a surface area of 3 m². Solar radiation is focused on the orifice of furnaces. Orifice diameter is little larger than that of the image of sun produced by mirror. Energy received is a little more than 2 kw when solar radiation intensity is 1 kw/m² (this is intensity of solar radiation at Mont-Louis, altitude 1600 meters).

Descriptions of furnaces of various capacities are given. A table summarizes maximum observed and calculated temperatures in various furnaces.

A. D. Kafadar, USA

1410. **Yeh, Tu Cheng, The motion of tropical storms under the influence of a superimposed southerly current,** *J. Meteor.* **7,** 2, 108–113, April 1950.

Author calculates for first time the 24-hour displacement of a hurricane (a) in an easterly trade current, and (b) as this current is affected by a disturbance in middle latitudes. The hurricane model chosen necessarily involves several assumptions. These, however, are sufficiently realistic to make paper of interest as a first approximation.

Hurricanes move with the mean speed of the trade and also execute inertia oscillations about the mean path which is in direction of the trade. When a southerly current is superimposed due to interference from middle latitudes, the hurricane accelerates northward but at rates varying on the initial conditions. Final storm positions after 24 hr for a given intensity of the southerly flow may be 200 km and more apart—a significant distance from the practical viewpoint.

H. Riehl, USA

1411. **Best, A. C., Empirical formulae for the terminal velocity of water drops falling through the atmosphere,** *Quart. J. roy. met. Soc.* **76,** 329, 302–311, July 1950.

Author fits a formula of the type $V = A \exp bz \{1 - \exp [1 - (d/a)^n]\}$ to experimental data on the terminal velocities of water drops ranging in diameter from 0.05 to 0.3 and 0.3 to 6.0 mm. Here, V is terminal velocity, z height, d diameter, or

diameter of sphere of equivalent volume, and A, b, a, and n are constants. For diameters less than 0.05 mm, the drops are in the Stokes region and $V = Ad^2 \exp bz$. Values of the constants and probable errors are given for the three ranges of drop size for both the I.C.A.N. standard atmosphere and the standard summer tropical atmosphere. Phillip Eisenberg, USA

1412. **Martyn, D. F., Cellular atmospheric waves in the ionosphere and troposphere,** *Proc. roy. Soc. Lond. Ser. A*, **201,** 1065, 216–234, Mar. 1950.

A theoretical study of moving electron clouds in ionosphere shows that these phenomena can be treated by same type of perturbation analysis ordinarily applied by meteorologists to wavelike disturbances in the troposphere. Hydrodynamic equations of motion are linearized by assuming small perturbations in the velocity and pressure fields, and are treated by assuming a linear dependence of wind and temperature upon elevation. Waves that are periodic with time and horizontal distance are considered. Although complete solution to perturbation equations thus derived is not obtained due to mathematical complexity, some physically significant frequency relations are developed. These consist in determining the frequency of oscillations which are bounded vertically, i.e., so that their energy does not leak away from layer in question, and discovering what atmospheric conditions give rise to formation of such boundary layers. It is found that bounding can be achieved either by an appropriate distribution of wind speed with elevation, or by a discontinuity in temperature or temperature gradient, being most sensitively dependent on the latter factor.

While study was originally undertaken to study ionospheric disturbances, work is shown also to apply to microbarometric oscillations in troposphere where periods of 10–13 min are shown to be adequately bounded. Application of theory to ionosphere is also supported by data, and it appears likely that previously observed "vertically" moving ion clouds are due merely to a phase advance with elevation in horizontally moving waves, due to interaction of the charged particles with the earth's magnetic field. It is concluded that for these ionospheric waves to be adequately bounded, $\gamma(c_p/c_v)$ must be less than 1.4. For those regions where atmospheric gases are dissociated, this conclusion is shown to be plausible.

Reviewer believes paper to be important because it is one of the first successful and observationally well-documented attempts to study ionospheric disturbances by techniques used in the meteorology of the troposphere. Joanne Starr Malkus, USA

Lubrication; Bearings; Wear

(See also Revs. 1000, 1202)

1413. **Macks, E. Fred., and Nemeth, Zolton N., Investigation of 75-millimeter-bore cylindrical roller bearings at high speeds. I,** *Nat. adv. Comm. Aero. tech. Note* 2128, 54 pp., July 1950.

Paper contains new and substantial information on such practical aspects as wear, roller slip, and circumferential distribution of temperature in jet-lubricated cylindrical roller bearings as applied in a conventional aircraft gas-turbine engine. In the test rig, significant differences as to the operating characteristics mentioned were found between the three standard types of bearings tested; these characteristics could partially be checked with those found in actual engine. Notwithstanding that roller slip occasionally attained high values, especially at high speeds and low loads, roller wear was negligible as long as failure did not occur. This fact is ascribed to the hydrodynamic formation of an oil film that the authors, inspired by the theory of Büche [*VDI-Forschung.* **5,** 237–244, 1934], postulate to exist between the rollers and their raceways. Reviewer cautions that Büche's theory needs revision in order to account for the equilibrium of the rollers.

H. Blok, Holland

1414. **Lawrence, K. B., A mathematical evaluation of pressures in a grease-lubricated bearing,** *Trans. Amer. Soc. mech. Engrs.* **72,** 4, 409–413, May 1950.

Author applies Christopherson's treatment [*Proc. Inst. mech. Engng.* **146,** p. 126, 1941] to a case of a journal bearing lubricated with grease. Treatment, based on curves connecting viscosity with rate of shear, makes no allowance for a yield value or for any separation of components of grease. Problem is simplified by assuming that boundary conditions are zero pressure at edges of a finite bearing covering an arc of 249°. Further simplification is achieved by assuming a constant mean film temperature.

Correspondence of form of calculated curves with those achieved by Cohn and Oren [title source, **71,** p. 555, 1949; AMR **3,** Rev. 2835] is not conclusive evidence of the power of the method because it is not possible to check angular correspondence of the points of maximum pressure. Moreover, the clearance assumed for calculation purposes differs by a factor of over two from that used in the experimental work referred to. It is regrettable that the calculations relate to a single speed of rotation, as Cohn and Oren report virtual constancy of pressure for a five-fold speed variation. Reduction of viscosity with shear rate may possibly compensate for increase in speed leading to a virtually constant value of the Sommerfeld variable. This should be explored as it has important technological implications.

While simplification of assumptions may prevent the treatment from being exhaustive, it is welcomed as a first attempt to solve a difficult but important problem. F. T. Barwell, Scotland

1415. **Johnson, Robert L., Godfrey, Douglas, and Bisson, Edmond E., Friction of surface films formed by decomposition of common lubricants of several types,** *Nat. adv. Comm. Aero. tech. Note* 2076, 28 pp., Apr. 1950.

The effect on friction of decomposition films formed by heating various petroleum and synthetic lubricants in contact with a steel surface is studied, using an apparatus employing an elastically restrained spherical rider moving in a spiral path on a rotating disk. The decomposition films cause significant reductions in friction. A thin film of the decomposition product of a silicone polymer in conjunction with the undecomposed liquid was particularly effective in lowering friction. Authors believe these low values are due to low shear strength of the film rather than hydrodynamic action of a very viscous film.

Milton C. Shaw, USA

1416. **Wannier, Gregory H., A contribution to the hydrodynamics of lubrication,** *Quart. appl. Math.* **8,** 1, 1–32, Apr. 1950.

Paper is composed of three sections. Section 1 deals with a new derivation of the well-known Reynolds equation. This arises from the Stokes equations in first approximation if all quantities entering latter are expanded in powers of the film thickness. Simple formulas are expanded giving such unknowns as the transverse pressure gradient from the Reynolds solution. The standard assumption that latter is zero is superfluous. In section 2, pressure distribution, resultant load, torque, and coefficient of friction are calculated for the closed infinite cylindrical bearing (no side leakage). Starting from Stokes equations, author applies the known electrostatic solution of the Laplace equation to solving the problem of flow between two eccentric circles. A comparison with Reynolds limit of very thin oil film shows that the differences are not very marked. Section 3 deals with the closed spherical

bearing with side leakage. Solution of the exact mathematical problem being too difficult, pressure, resultant load, torque, and coefficient of friction are calculated in the Reynolds limit; this also has not been done up to this time. Analysis shows that side leakage does not play a very essential role in the theory of lubrication. All the main features are the same in the two cases: pressure maximum near the point of least clearance, region of negative pressure of the exit side, antisymmetry of the pattern. A factor of the order $^1/_2$ indicates the effect of side leakage on the pressure. W. Kochanowski, Germany

1417. Kahlert, W., The influence of the inertia forces in the hydrodynamic theory of film lubrication (in German), *Ingen.-Arch.* **16**, 5–6, 321–342, 1948.

The theory of film lubrication generally admits that inertia forces are negligible. Author develops a mode of calculation taking into account, to a degree, these forces for bearings with infinite width. The velocities, pressures, resultant of pressures, point of application of this resultant, friction and coefficient of friction are first calculated in the usual manner (viscosity constant) and are indicated with index 1 (u_1, p_1, P_1, T_1, and w_1). These values of velocities and their derivatives are then used for calculating inertia terms of the differential equation of Navier-Stokes, applicable to plane motion of incompressible fluids. Correction values can be secured for u and p (u_K, p_K) from the Navier-Stokes equation and from the equation of continuity, with convenient limiting values of u_K. The admitted values for velocity and pressure are then: $u_2 = u_1 + u_K$, $p_2 = p_1 + p_K$, and so on for other characteristics. This theory is applied to slide-in shaft bearings and the different correction values are found to be proportional to a number R_e^* (KR_e^*) connected with the Reynolds number. For the slide bearings, R_e is found to be equal to $(ul/\nu)(h/l)^2$ (h film thickness, l slide length).

The application point hardly changes. The coefficients of correction for the coefficient of friction are small and positive for values of $(h_1 - h_2)/h_2$ smaller than 2, and negative for larger values.

In calculations of the half closed bearings, pressures p_1, given by positive values of classical theory on the whole enclosed shaft, are used for determining inertia terms of the differential equation.

Correction factors for the pressures are small and generally positive except for small values of relative eccentricity and in vicinity of outlet. Direction of the pressure resultant is practically unchanged. Correction factors for the coefficient of friction (smaller than 2%) are positive for values of relative eccentricity smaller than about 0.5, and negative for larger values.

In his introduction, author gives some bibliography but mentions only the names of Reynolds and Michell of the English literature. The appreciation of influence of inertia forces on values of classical theory, for chosen example, seems too large (p. 324, no. 3). D. DeMeulemeester, Belgium

1418. Levitsky, M. P., On the temperatures of the friction surface of solids (in Russian) *Zh. teckh. Fiz.* **19**, 9, 1010–1014, Sept. 1949.

Temperature at rubbing solid surfaces is shown theoretically to depend on square root of relative speed, rather than directly on this speed, as in the work of Bowden. Discrepancy is due to effect of heat loss into the moving surface. By taking this effect into account, Bowden's formula is made to agree with that of author. The derived relationship has been experimentally verified during metal cutting. Walter W. Soroka, USA

Marine Engineering Problems

(See also Rev. 1028)

1419. Van Lammeren, W. P. A., State of the scientific research at the Netherlands Ship Model Basin in Wageningen (in Dutch), *Schip en Werf* **17**, nos. 16, 17, 357–363, 379–385, Aug. 1950.

Paper surveys the financial and staff organization of the department for scientific research of the Netherlands Ship Model Basin. It contains a statement on the progress of 16 subjects on the long-distance research program, mainly in the field of ship propulsion. A description is given of the extension of the known Wageningen systematic screw series to larger disk-area ratios, smaller pitch ratios, and two- and five-bladed propellers. Special propeller designs for tugs and icebreakers are discussed. The influence of cavitation on the thrust of propellers is treated, as well as the vibration of propeller blades. Other subjects included are the influence of air content of the water on burbling and sheet cavitation, and cavitation erosion. The development of constructional principles for wake-adapted propellers and the application of profiles of shockfree entrance in an inhomogeneous velocity field are described. Measurements on the characteristics of thin von Kármán-Trefftz profiles in cavitating condition, carried out in a special flume built in the cavitation tunnel, are well under way.

Paper is generally descriptive, but reviews very interesting research activity in the ship-propulsion field.

L. Troost, Holland

1420. Castagneto, E., The screw propeller with adjustable blades in its hydrodynamic behaviour (in Italian), *Riv. maritt. Suppl. tech.*, 49–68, July 1948.

1421. Büchi, G., The shell as improvement of the helical propeller (in Italian), *Riv. maritt. Suppl. tech.*, 7–47, July 1948.

1422. Strandhagen, Adolf G., Schoenherr, Karl E., and Kobayashi, Francis M., The dynamic stability on course of towed ships, *Soc. nav. Arch. mar. Engrs.*, no. 21, 15 pp. May 1950.

Towing of an unpowered vessel by a powered vessel by means of a towline may become very difficult and even dangerous when the vessel tows unstably. Stability is treated by the known method of examining the dynamical consequence of an infinitesimal disturbance from equilibrium. The novel feature is a thorough investigation of the effect of an additional parameter, ratio of towline length to length of ship.

Stability analysis yields an equation of fourth degree for the so-called "stability index." Routh's criteria are then applied to discuss stability. Method is illustrated by application to a towed destroyer and a towed minesweeper. An alternative method of investigating stability by means of graphs of the roots of the stability equation against the towline-length parameter was suggested and illustrated by the reviewer in the written discussion on present paper. L. Landweber, USA

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